# Enabling Rubin Science with Robust Cross-Matches in the Crowded LSST Sky

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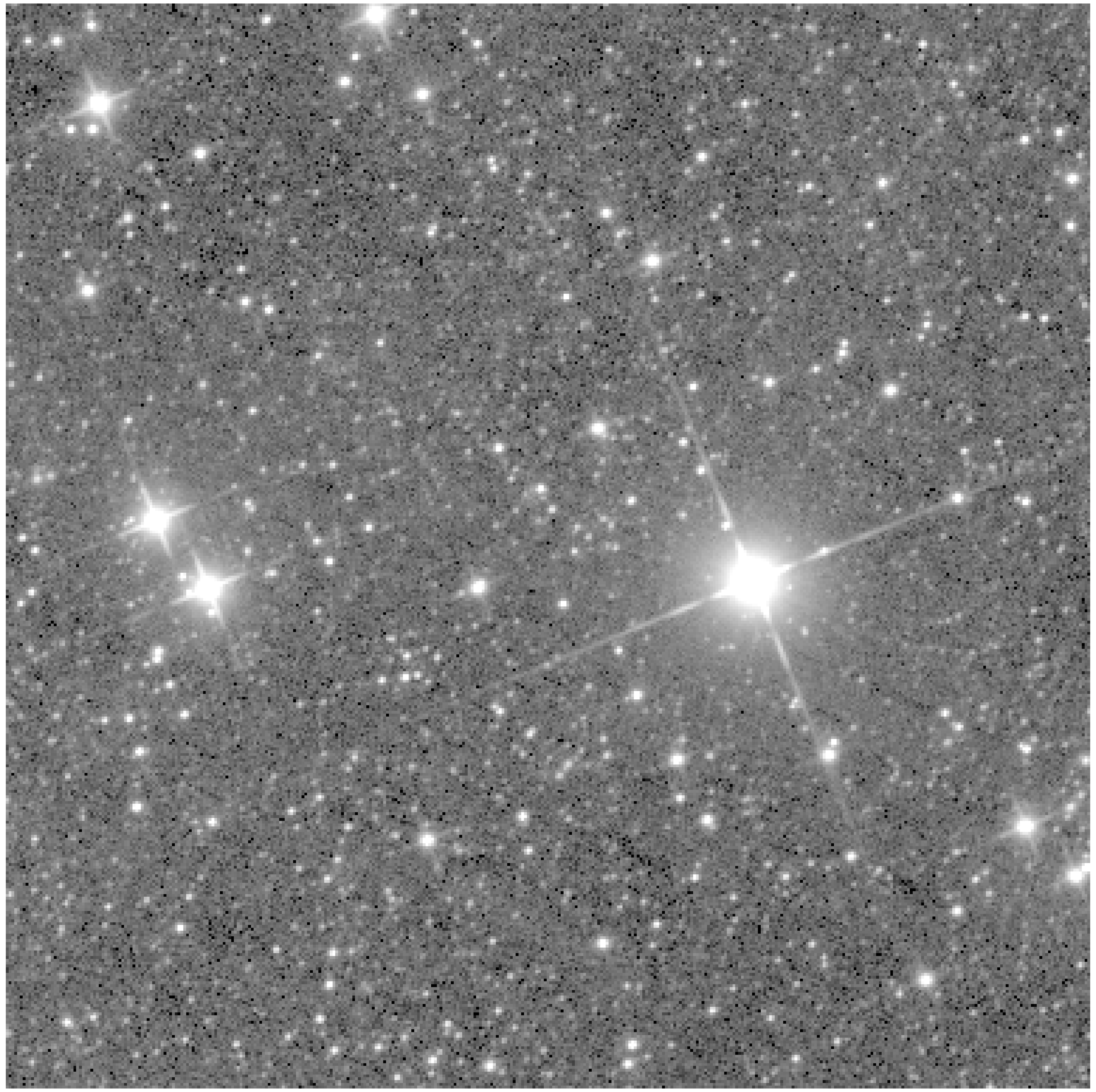
# Photometric Observations



*WISE* - Wright et al. (2010)

*WISE* W1

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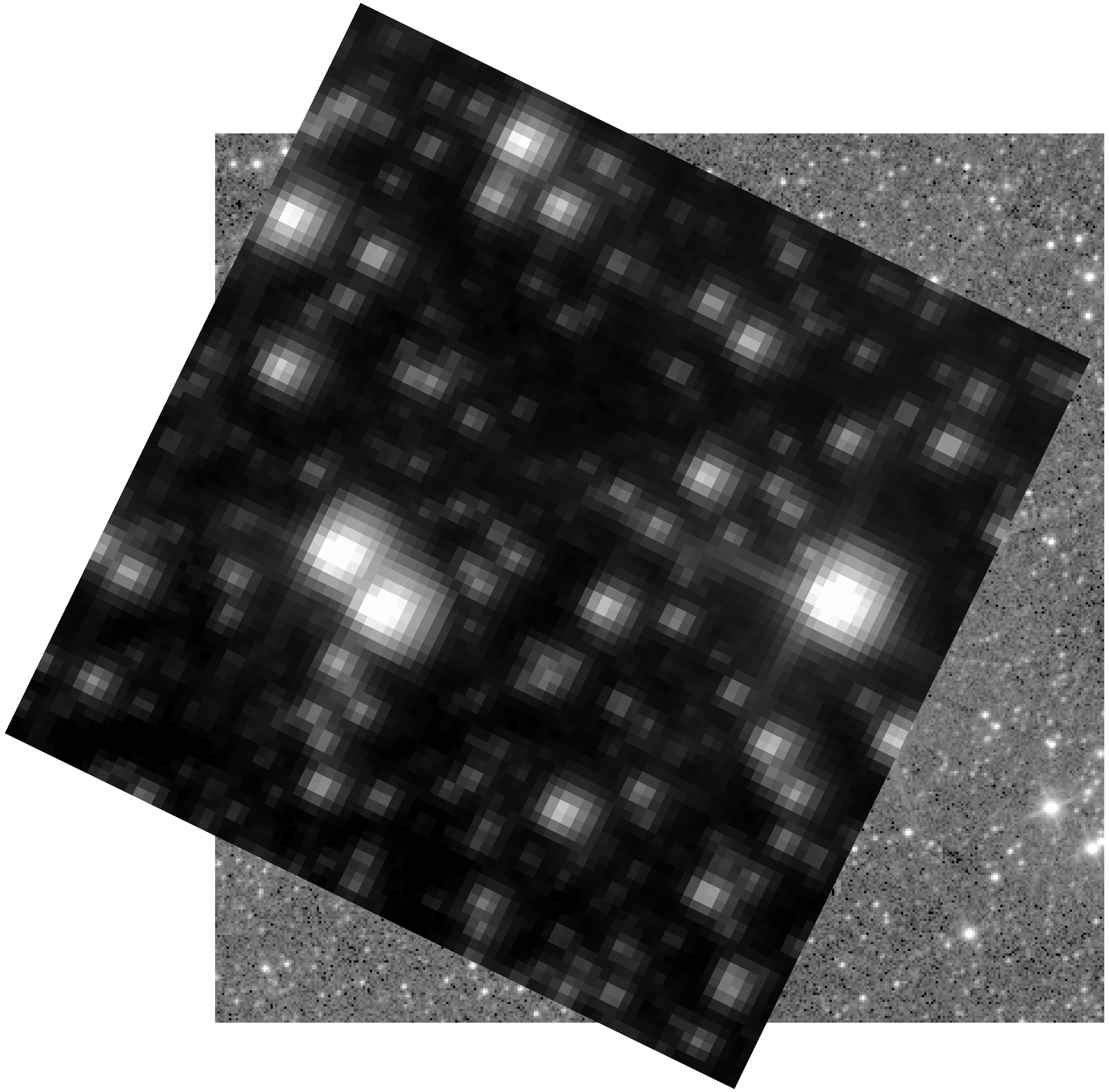
# Photometric Observations



*WISE* - Wright et al. (2010)
*TESS* - Ricker et al. (2015)

*TESS* T

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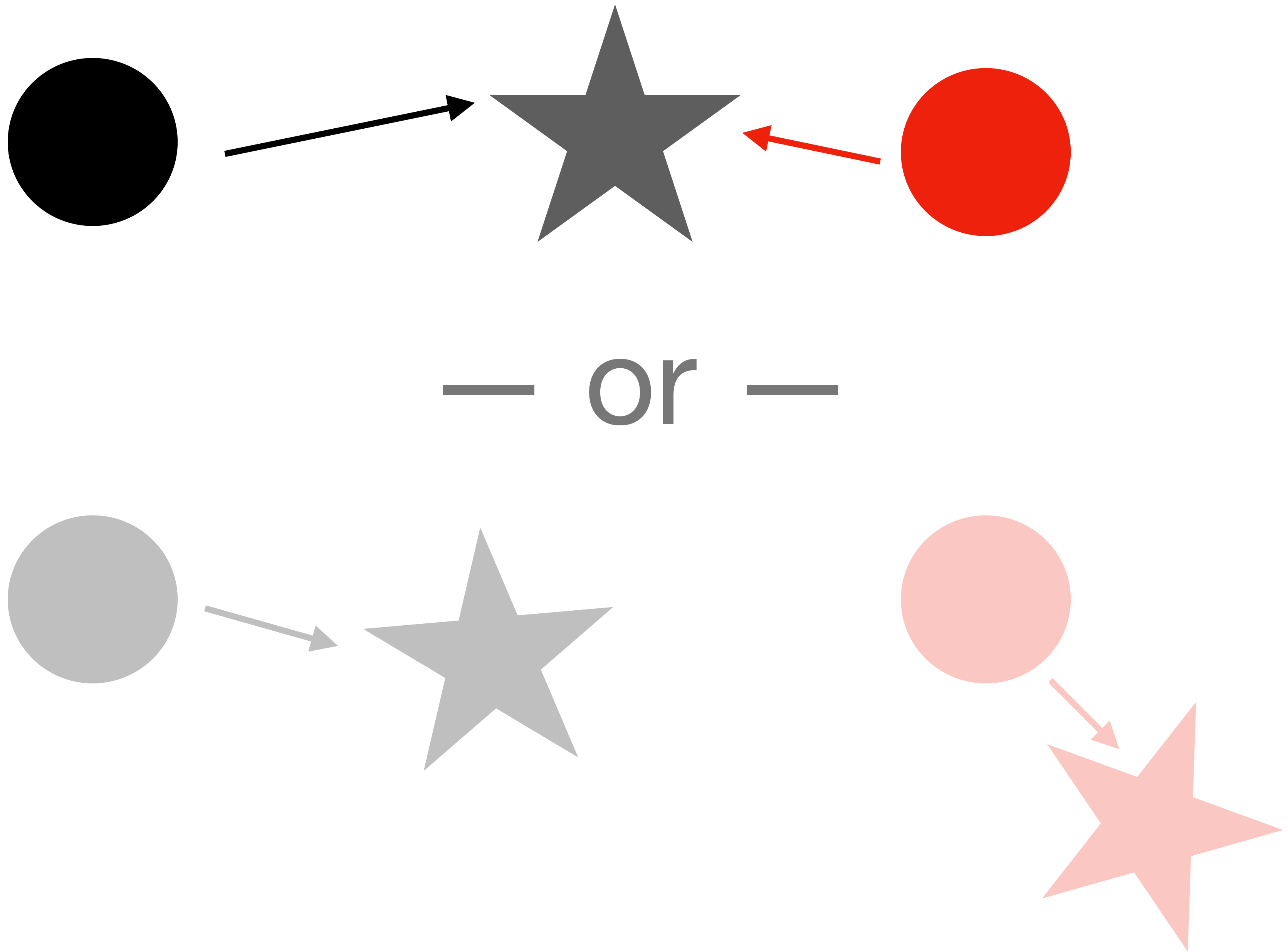
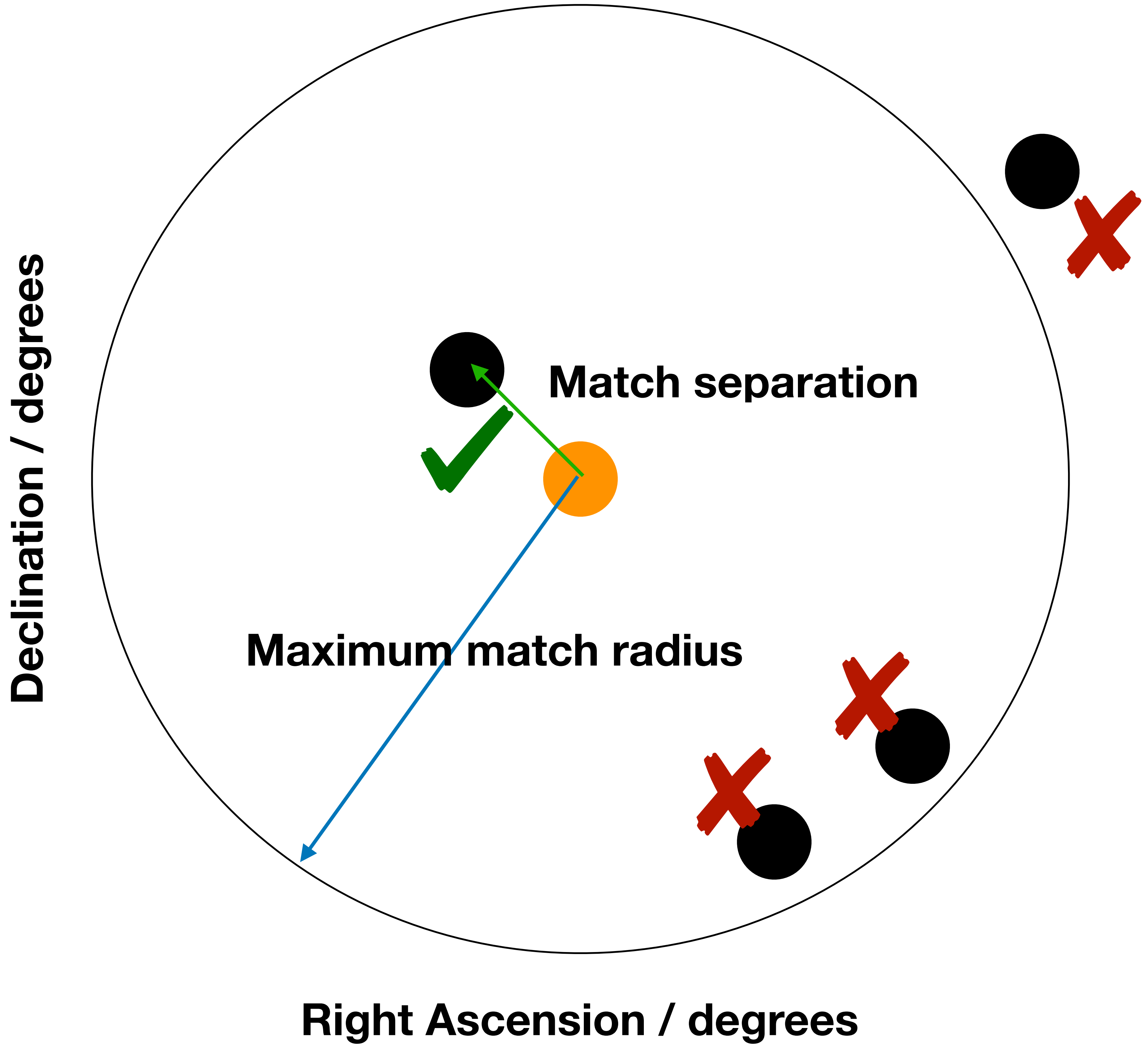
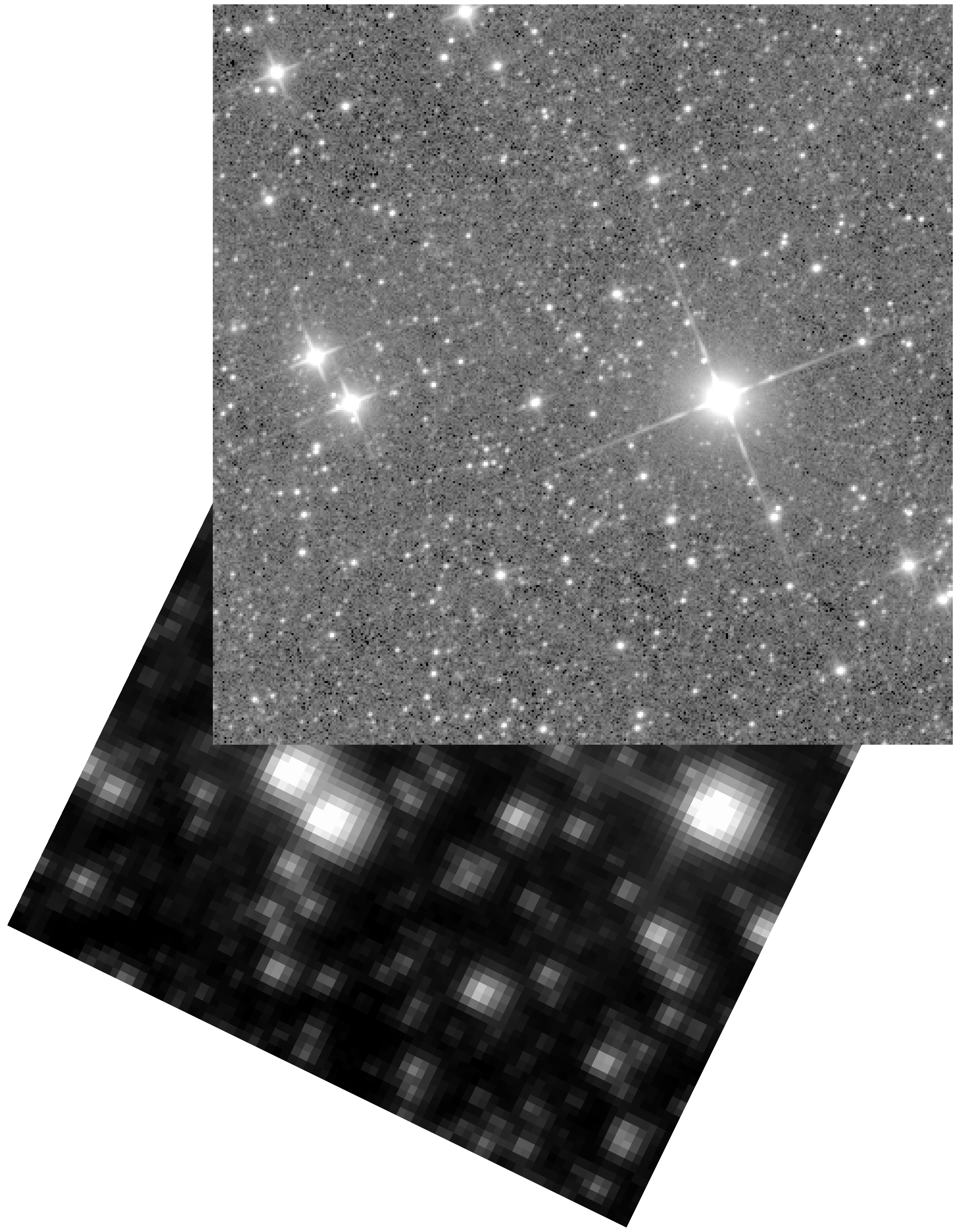
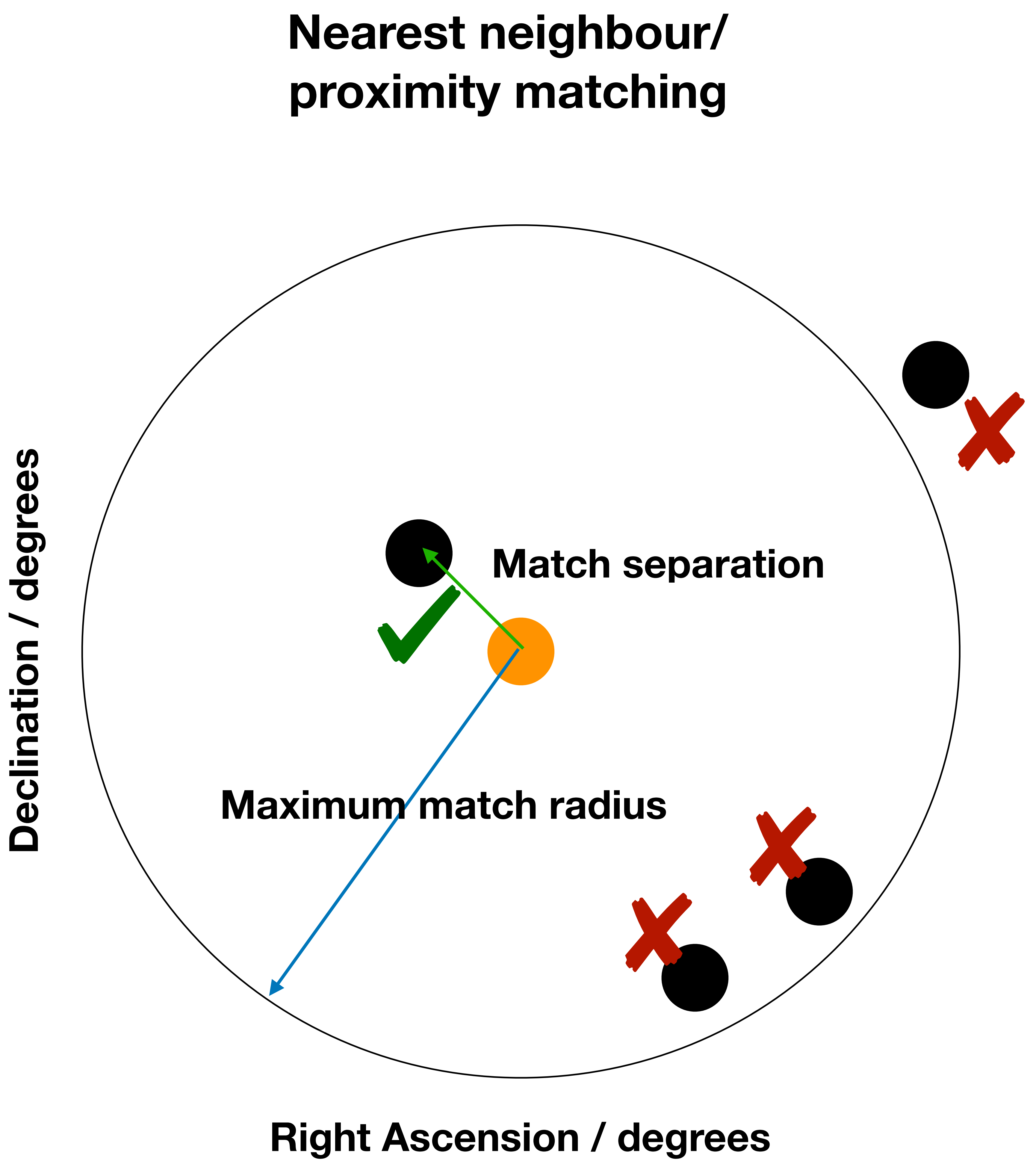
# “Simple” Cross-Matching

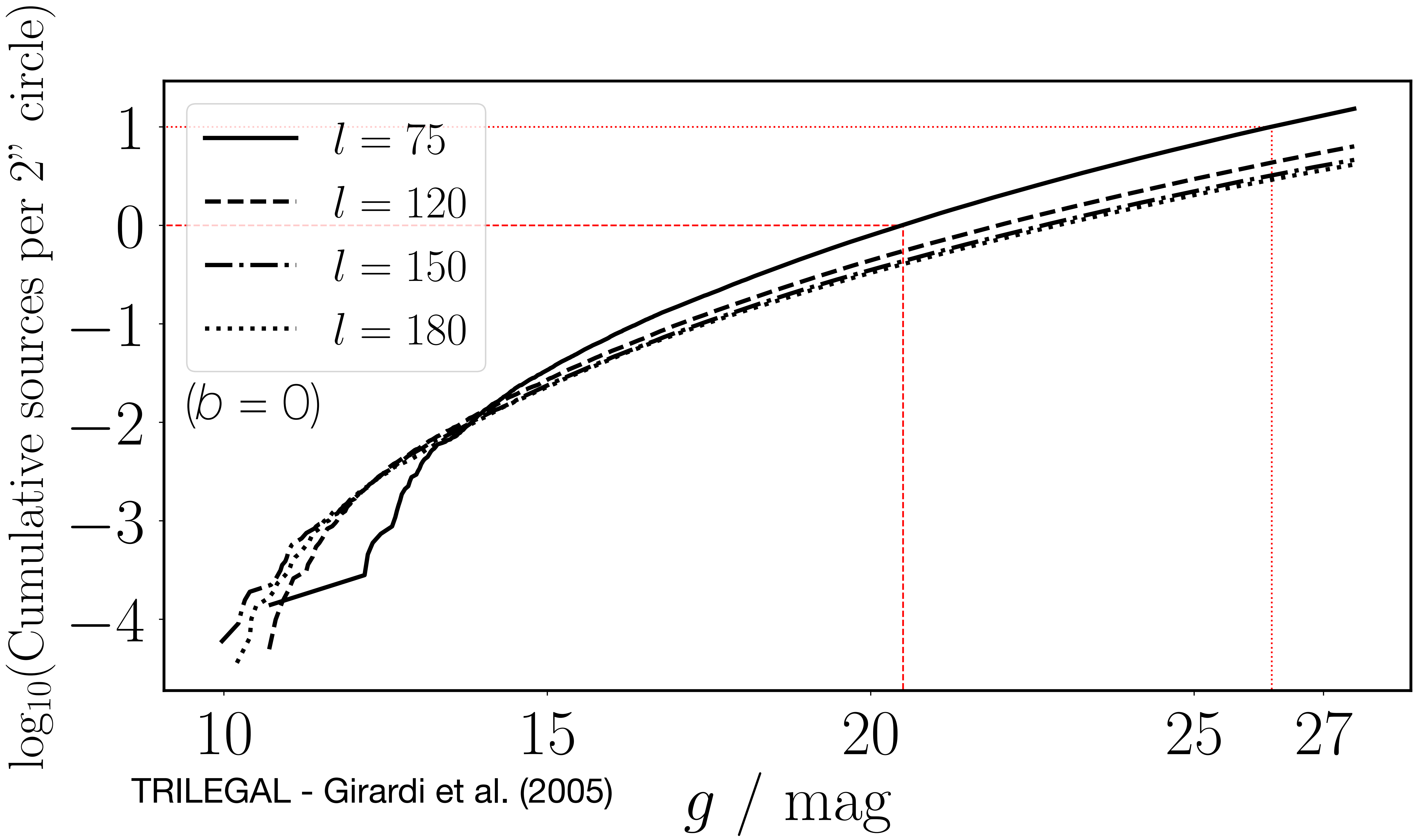


# The Problem With Rubin Obs.'s LSST




TRILEGAL - Girardi et al. (2005)

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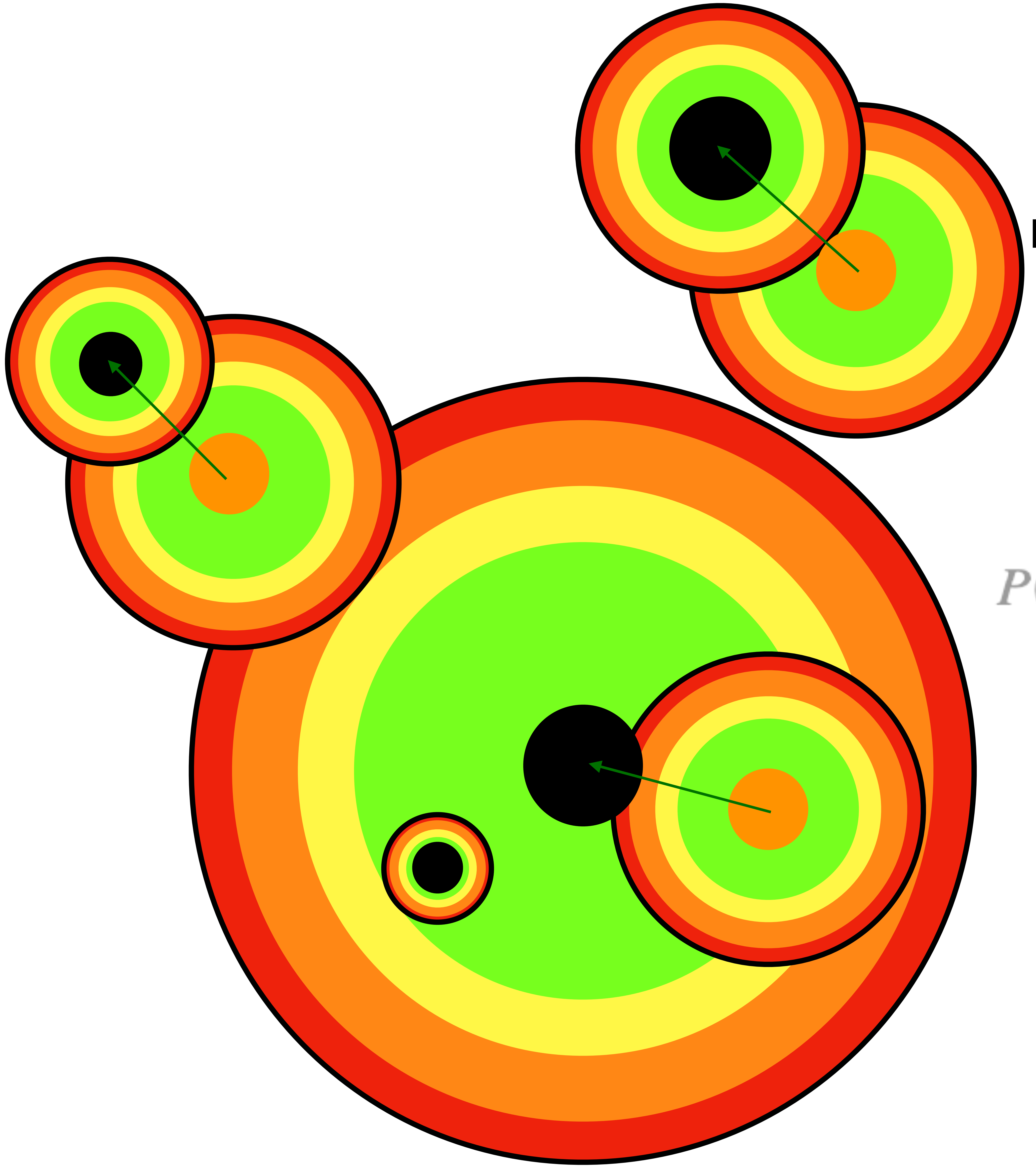
# Probabilistic Cross-Matching



Probability of two sources having their on-sky separation given the hypothesis they are counterparts

Probability of sources having their brightnesses given they are unrelated to one another (“field stars”)

Probability of sources having their brightnesses given they are counterparts

$$P(\zeta, \lambda, k|\gamma, \phi) = \frac{1}{K} \times \prod_{\delta\notin\zeta\cap\delta\in\gamma} N_\gamma \boldsymbol{f}_{\boldsymbol{\gamma}}^{\boldsymbol{\delta}} \prod_{\omega\notin\lambda\cap\omega\in\phi} N_\phi \boldsymbol{f}_{\boldsymbol{\phi}}^{\boldsymbol{\omega}} \prod_{i=1}^{k} N_c \boldsymbol{G}_{\gamma\phi}^{\zeta_i\lambda_i} \boldsymbol{c}_{\gamma\phi}^{\zeta_i\lambda_i}$$

Wilson & Naylor (2018a)

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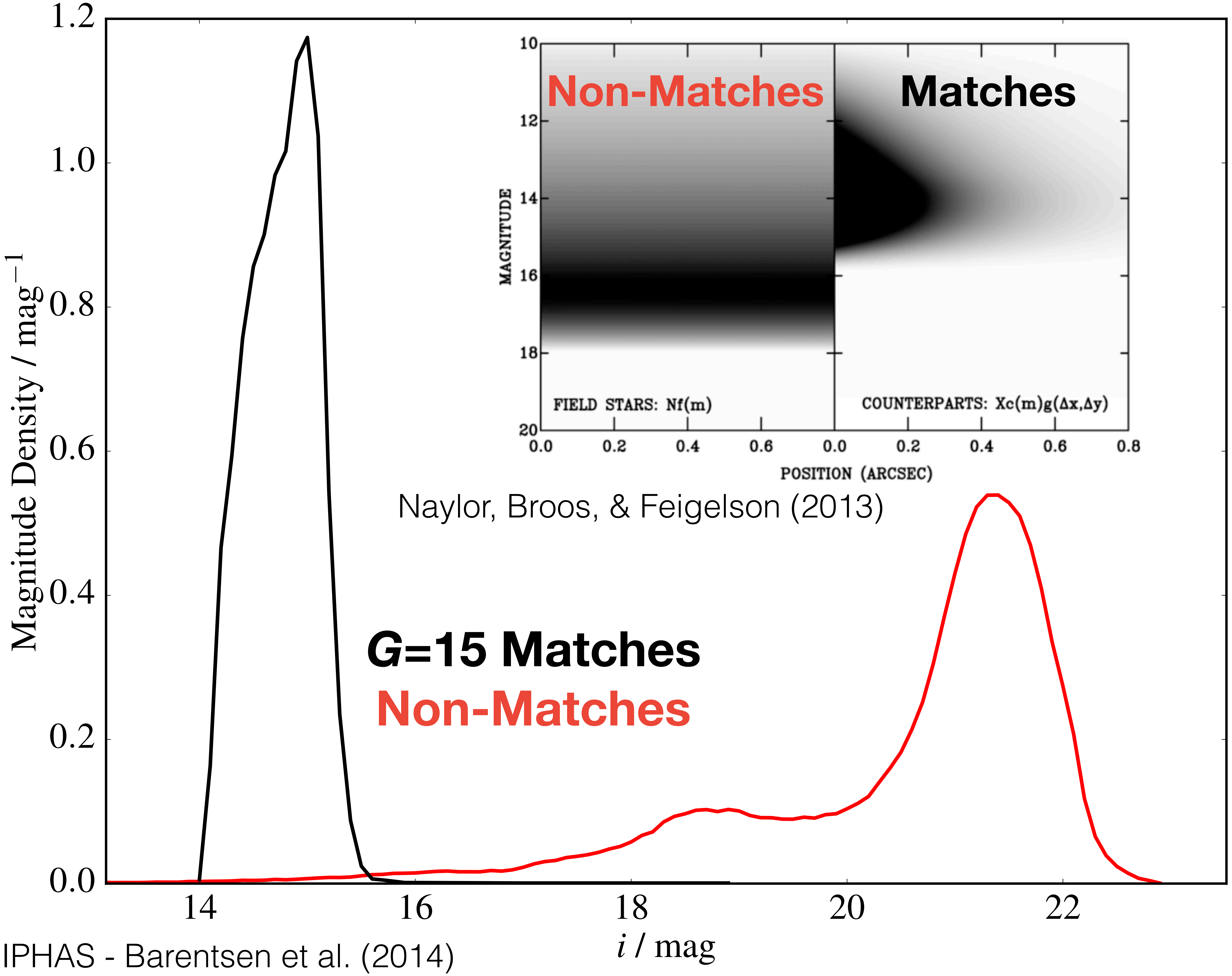
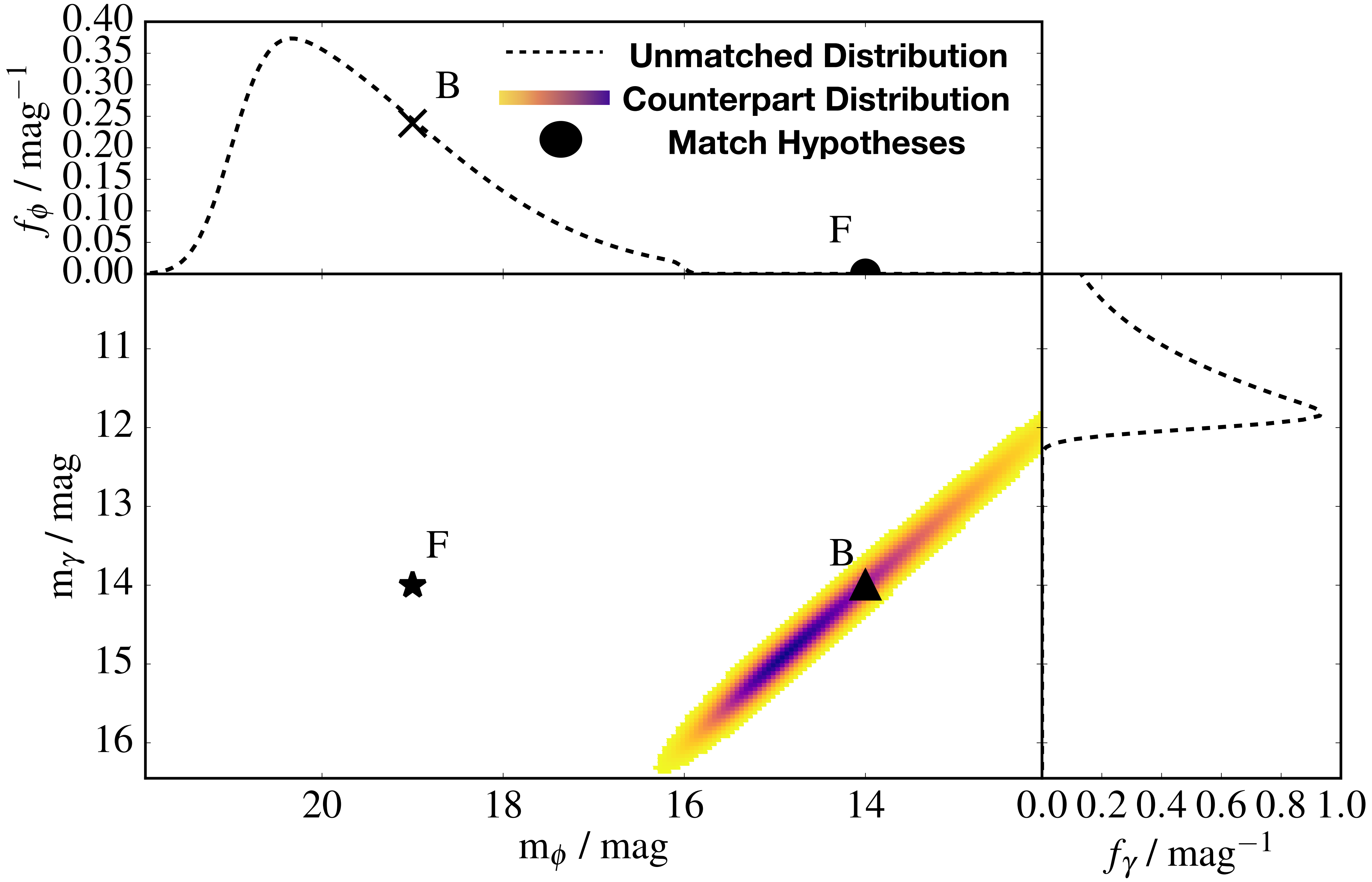
# Including Magnitude Information

$$P(\zeta,\lambda,k|\gamma,\phi) = \frac{1}{K} \times \prod_{\delta\notin\zeta\cap\delta\in\gamma} N_\gamma f_\gamma^\delta \prod_{\omega\notin\lambda\cap\omega\in\phi} N_\phi f_\phi^\omega \prod_{i=1}^{k} N_c G_{\gamma\phi}^{\zeta_i\lambda_i} c_{\gamma\phi}^{\zeta_i\lambda_i}$$



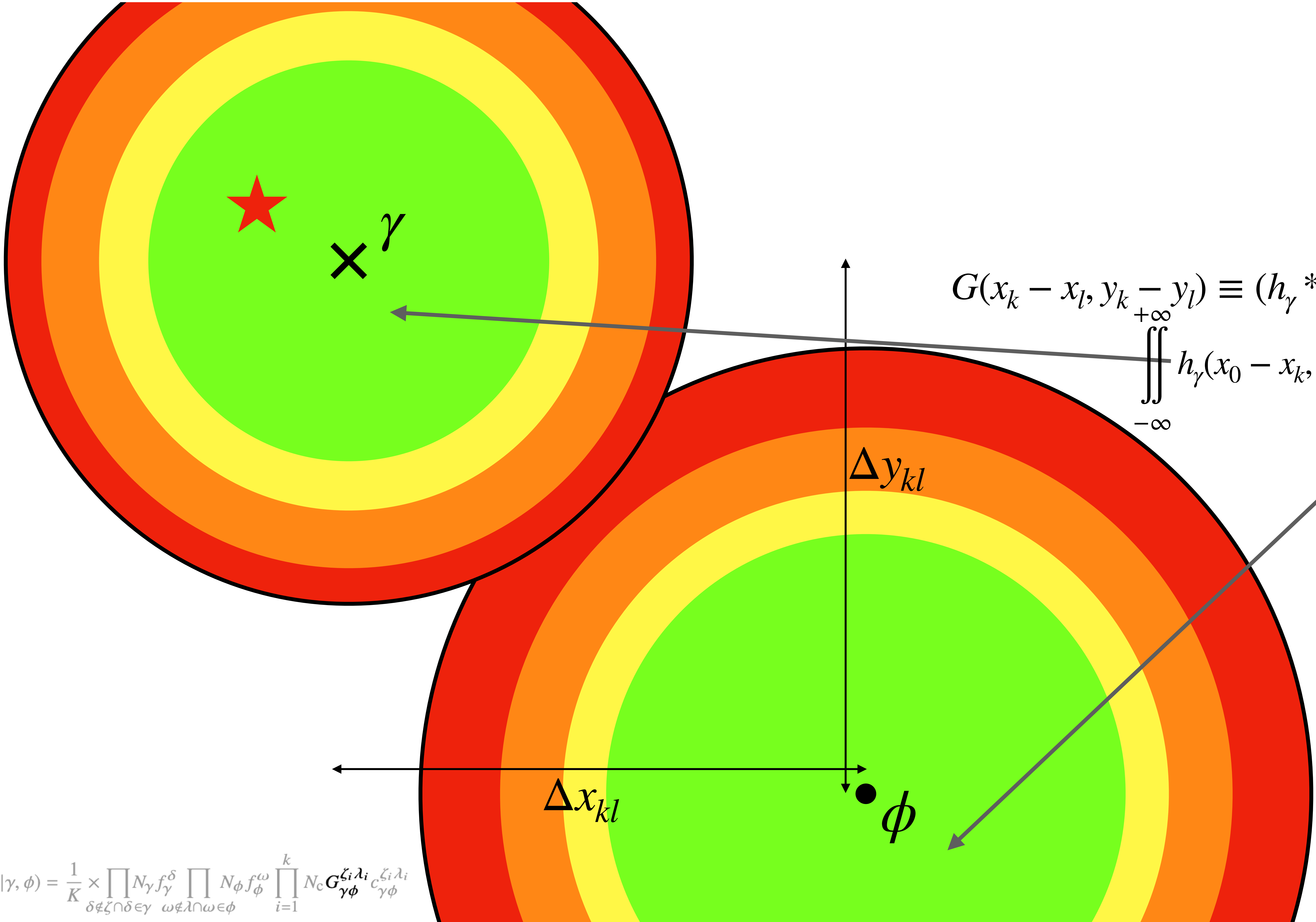

IPHAS - Barentsen et al. (2014)
Gaia DR2 - Gaia Collaboration, Brown A. G. A., et al. (2018)




Wilson & Naylor (2018a)

**The photometry-based likelihoods ($c$ and $f$) allow us to mitigate high false positive rate in crowded fields, but now we need the position-based likelihood $G$**

# Match Separation Probability




$$G(x_k - x_l, y_k - y_l) \equiv (h_\gamma * h_\phi)(\Delta x_{kl}, \Delta y_{kl}) = \iint\limits_{-\infty}^{+\infty} h_\gamma(x_0 - x_k, y_0 - y_k) h_\phi(x_l - x_0, y_l - y_0)\, \mathrm{d}x_0\, \mathrm{d}y_0$$

Wilson & Naylor (2018a)

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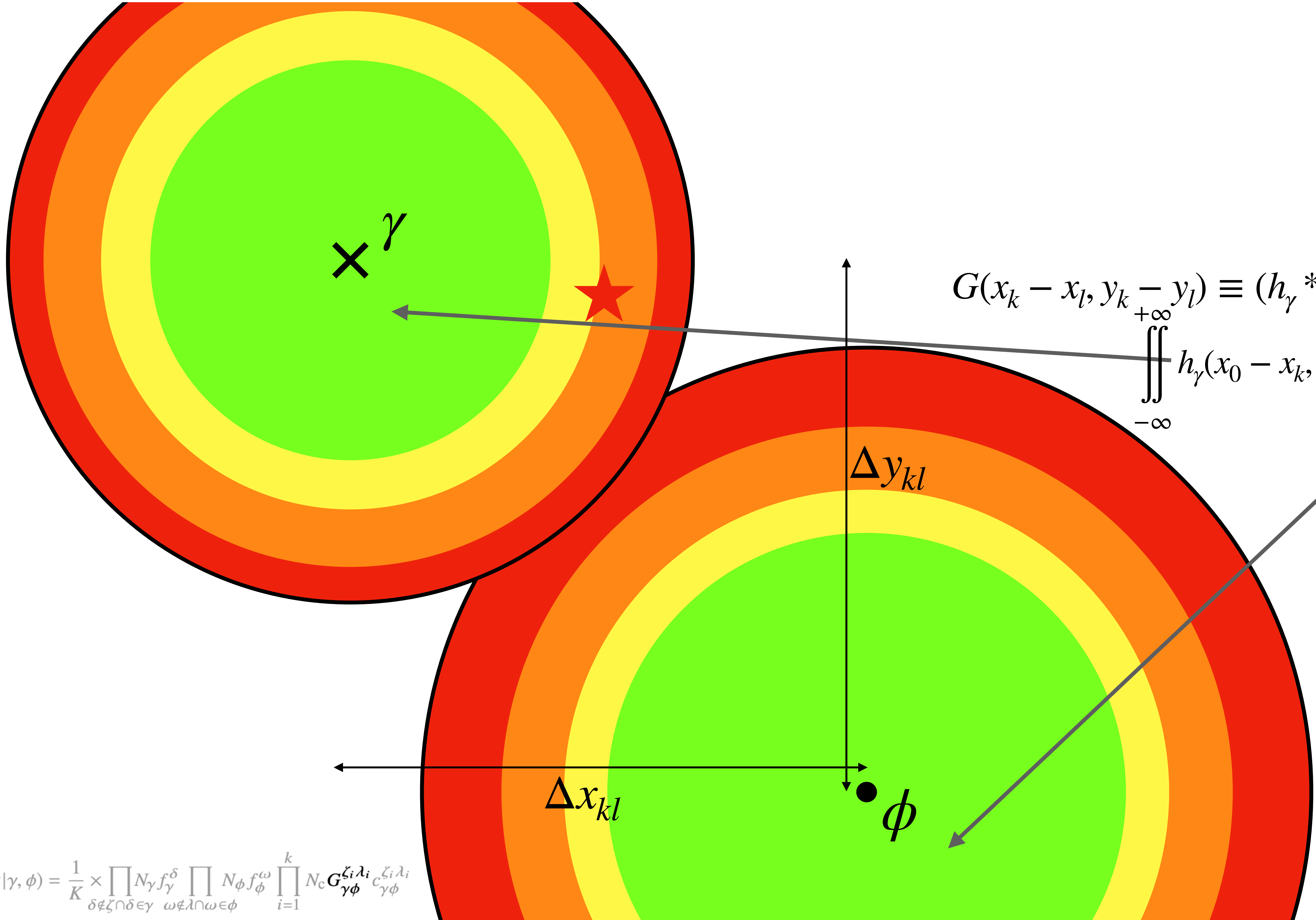


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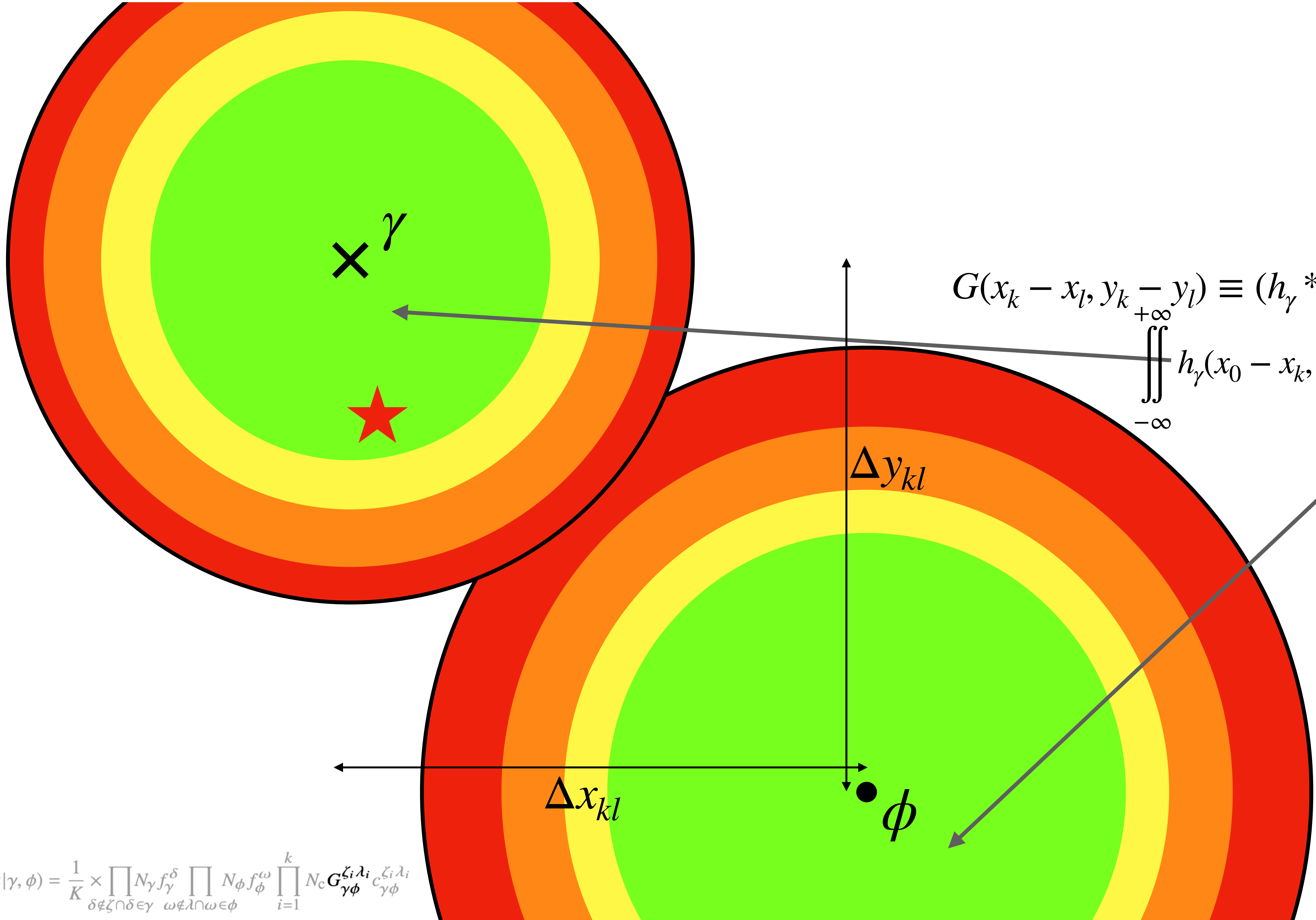


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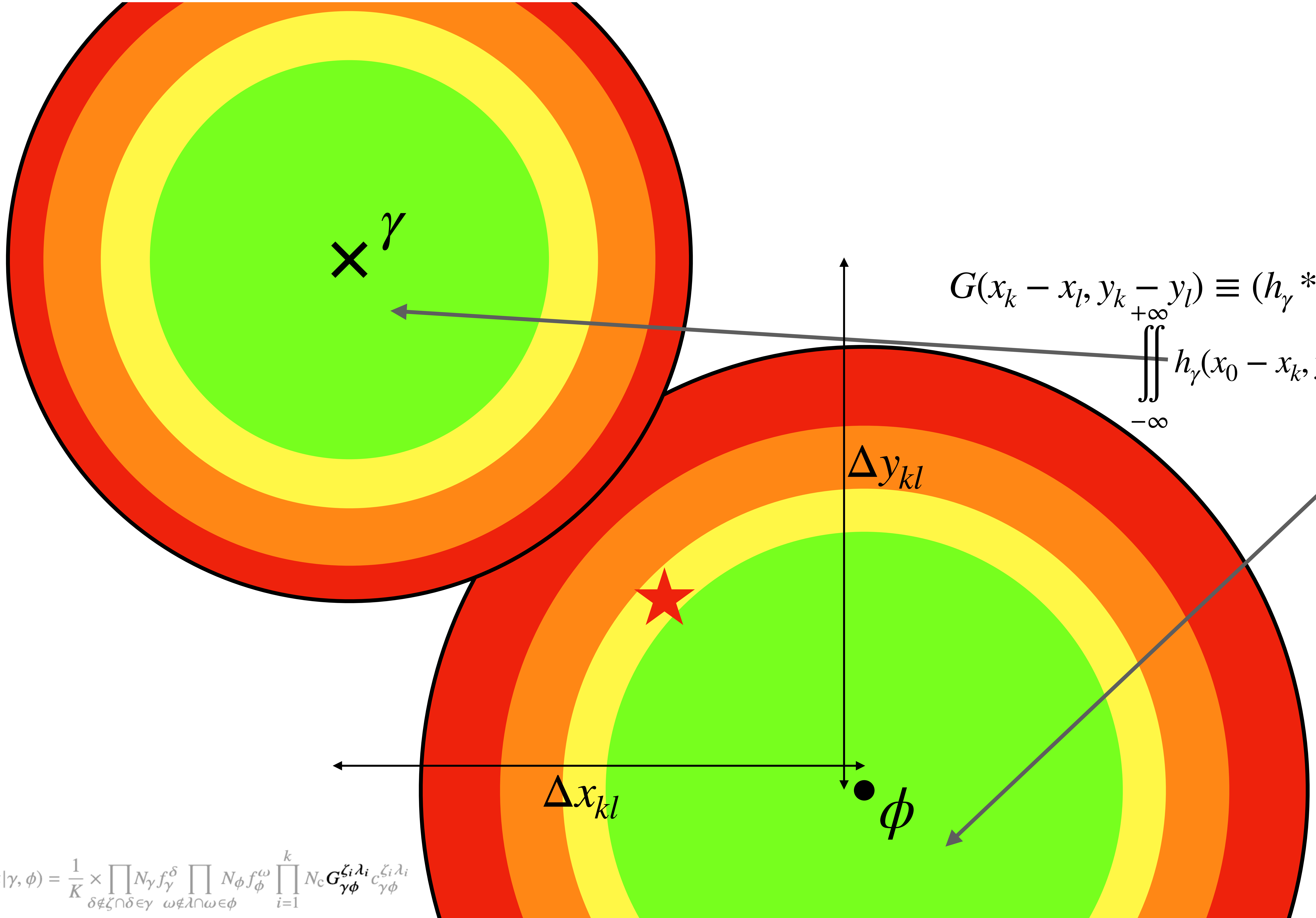


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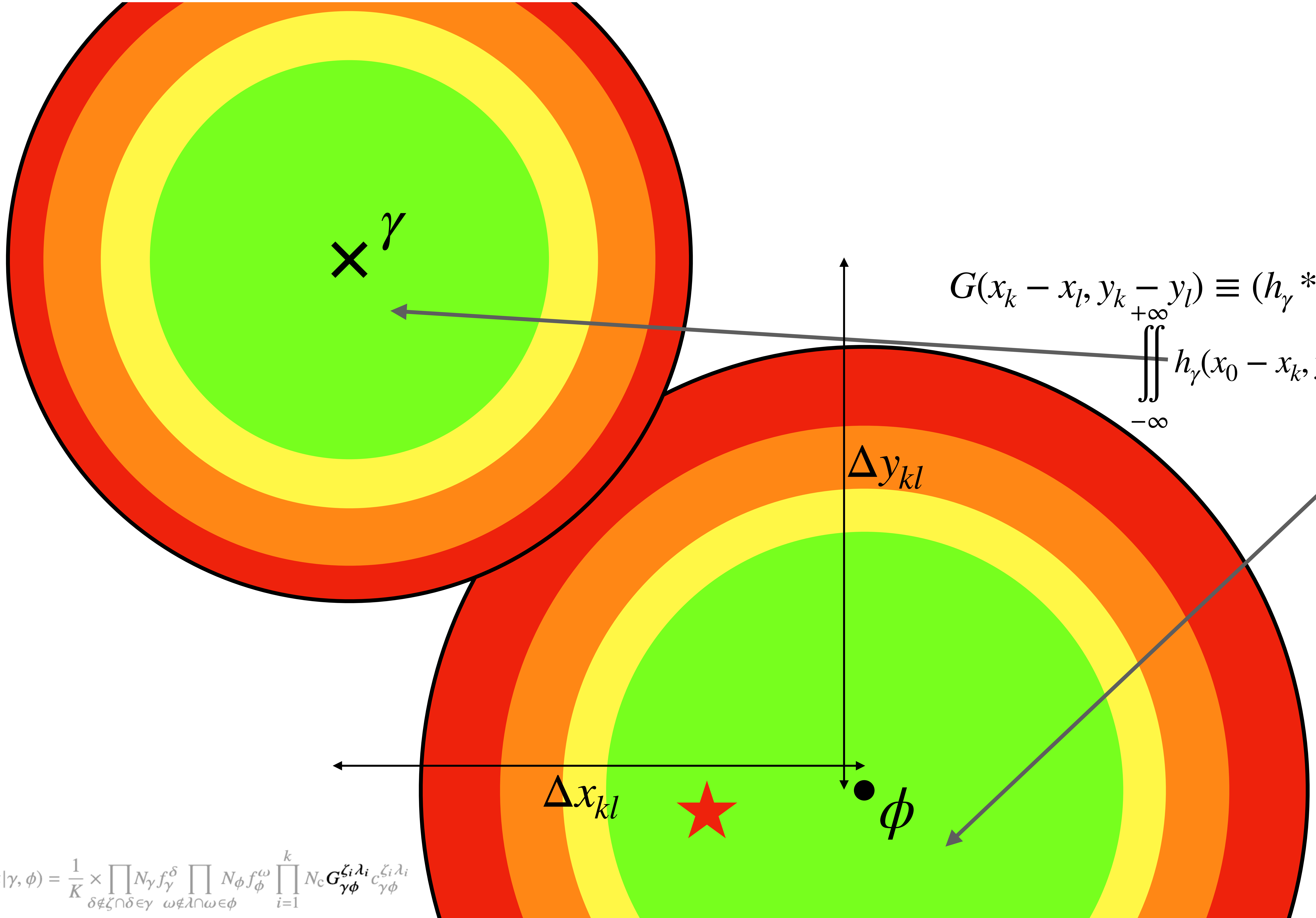


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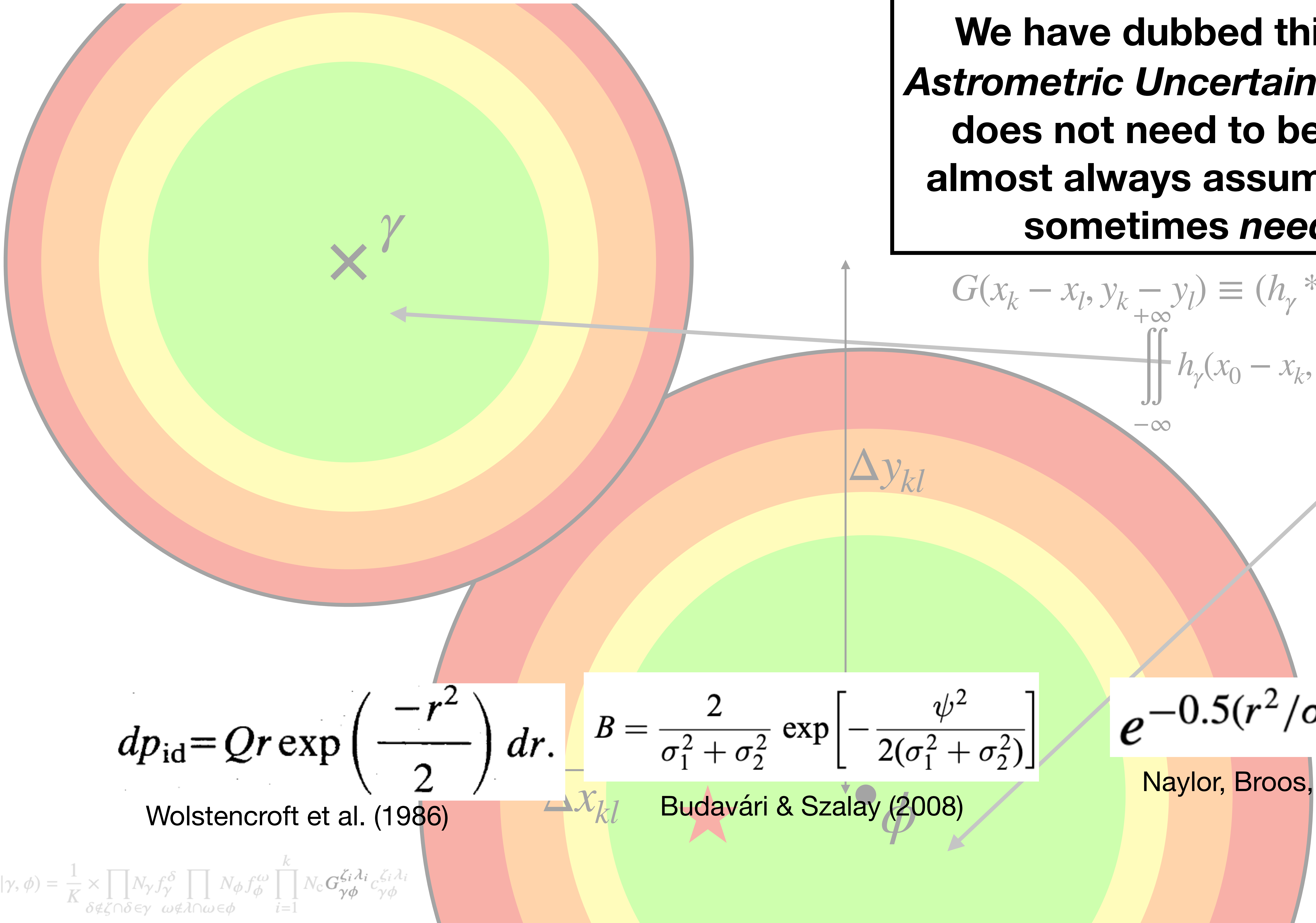
# Match Separation Probability

**We have dubbed this function $h$ the *Astrometric Uncertainty Function,* which does not need to be Gaussian, as is almost always assumed — and indeed sometimes *needs* not to be!**

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Wilson & Naylor (2018a)

$$dp_{\mathrm{id}} = Qr \exp\left(\frac{-r^2}{2}\right) dr.$$

Wolstencroft et al. (1986)

$$B = \frac{2}{\sigma_1^2 + \sigma_2^2} \exp\left[-\frac{\psi^2}{2(\sigma_1^2 + \sigma_2^2)}\right]$$

Budavári & Szalay (2008)

$$e^{-0.5(r^2/\sigma_{39}^2)}$$

Naylor, Broos, & Feigelson (2013)

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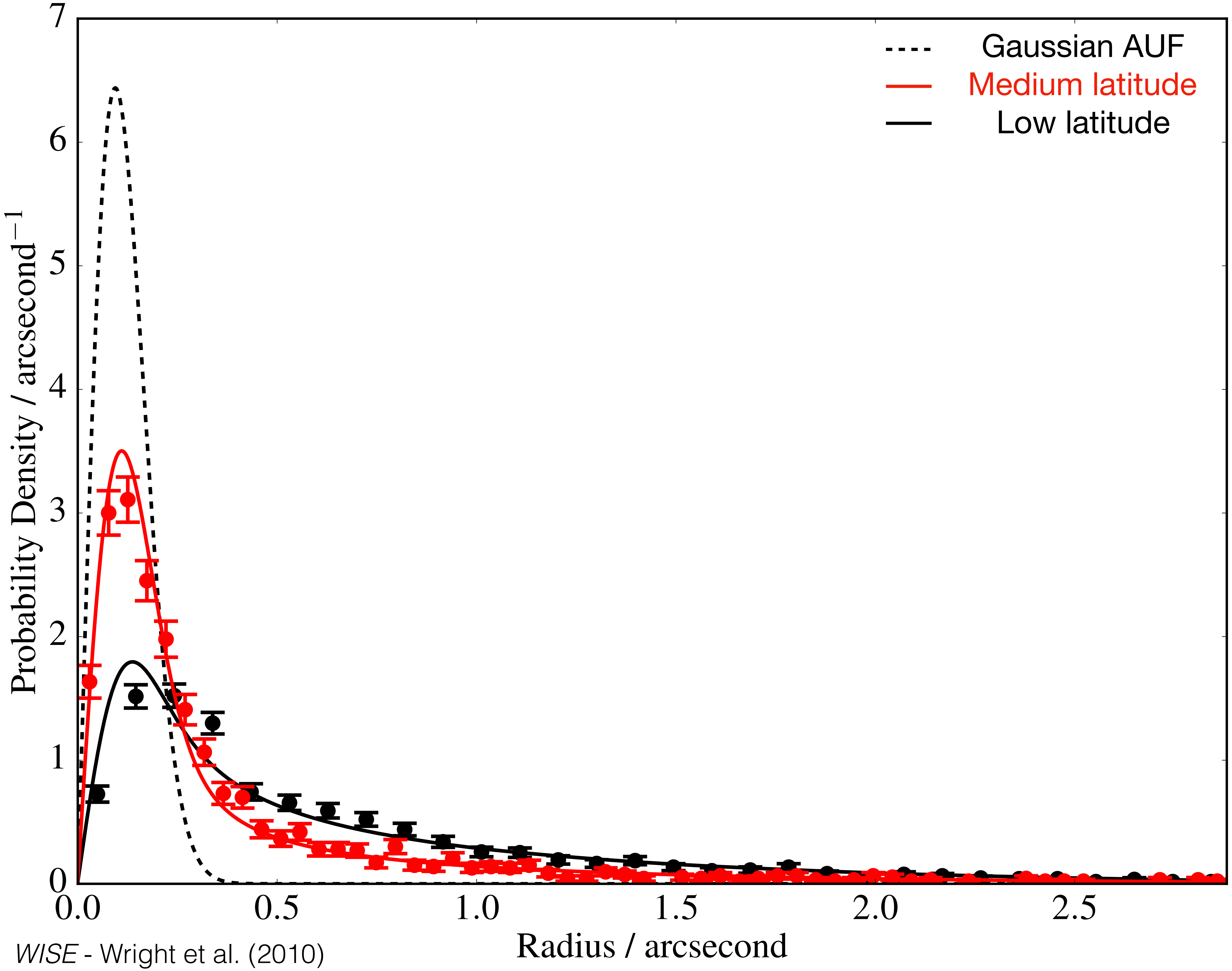
# Additional Components of the AUF

$$P(\zeta, \lambda, k|\gamma, \phi) = \frac{1}{K} \times \prod_{\delta \notin \zeta \cap \delta \in \gamma} N_\gamma f_\gamma^\delta \prod_{\omega \notin \lambda \cap \omega \in \phi} N_\phi f_\phi^\omega \prod_{i=1}^{k} N_c \boldsymbol{G}_{\boldsymbol{\gamma\phi}}^{\boldsymbol{\zeta_i \lambda_i}} c_{\gamma\phi}^{\zeta_i \lambda_i}$$




*WISE* - Wright et al. (2010)
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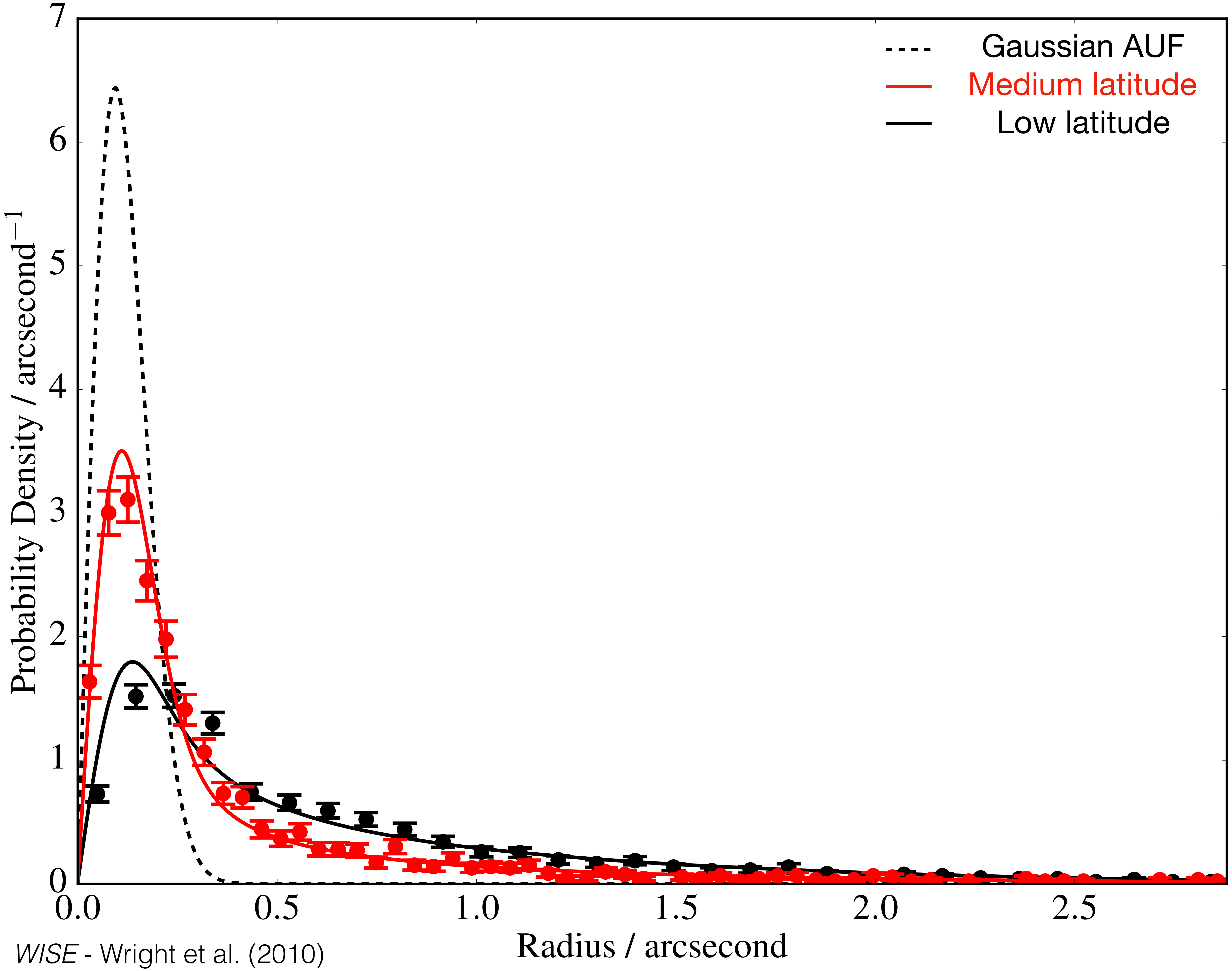


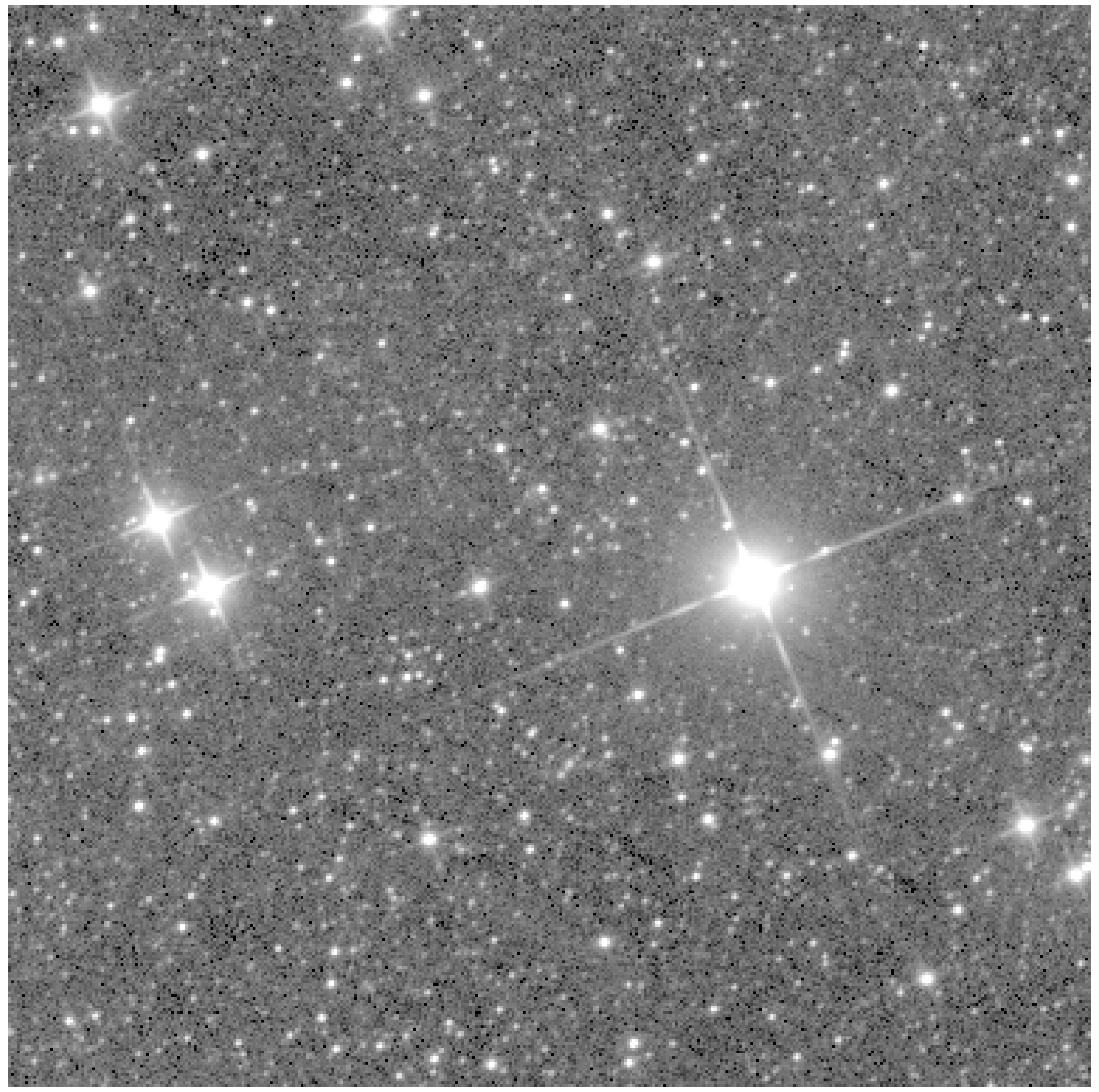

*WISE* - Wright et al. (2010)

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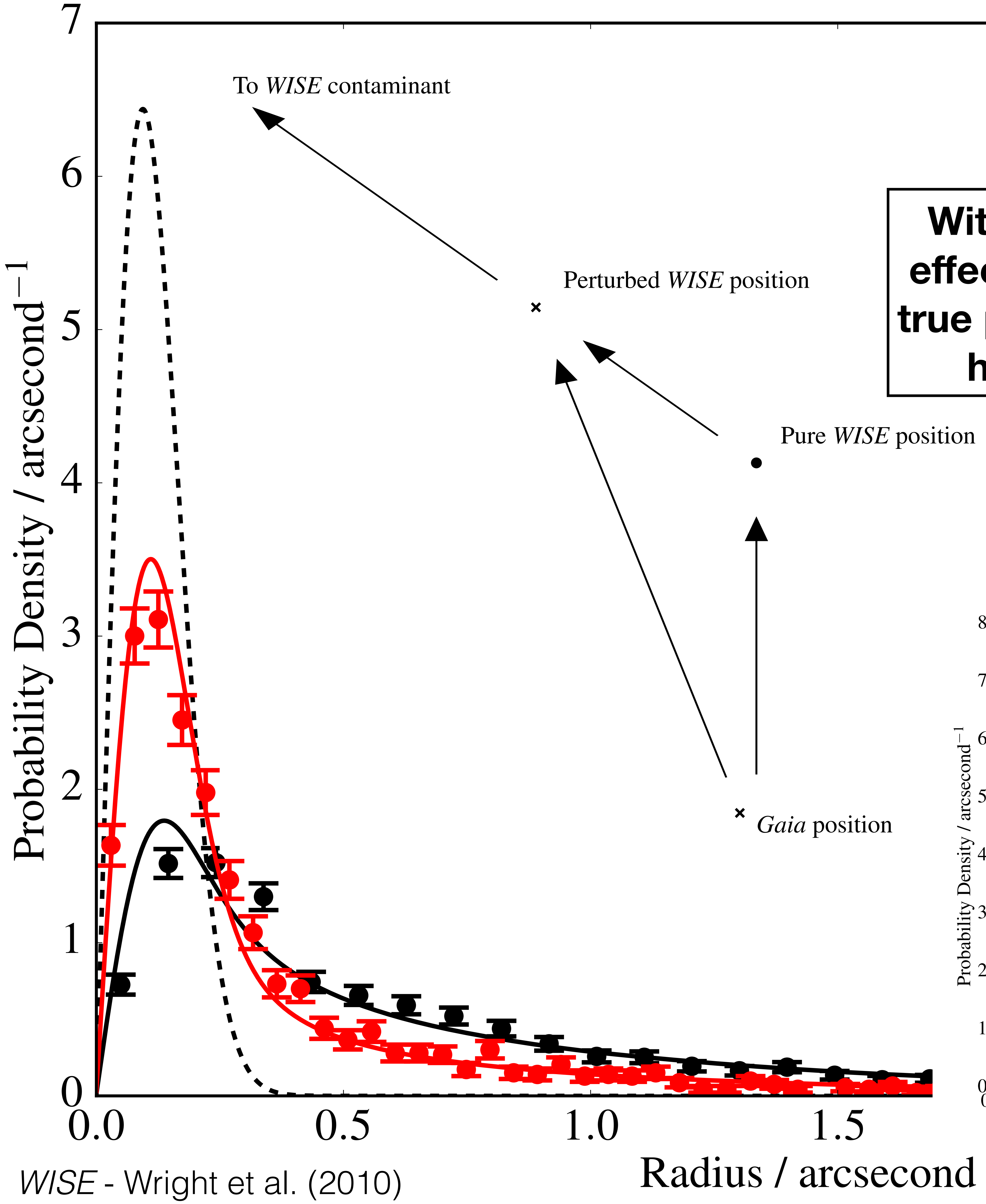
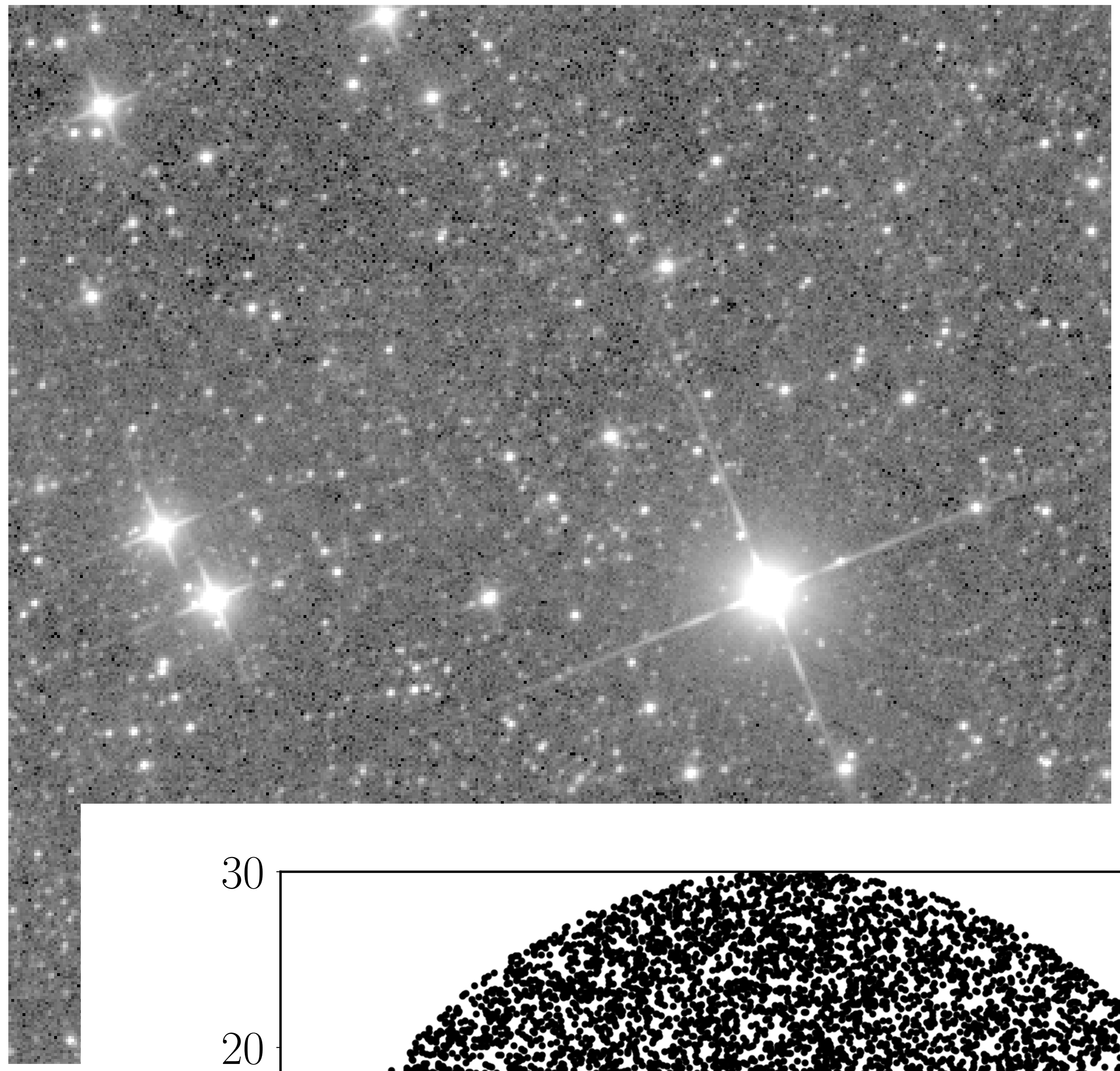
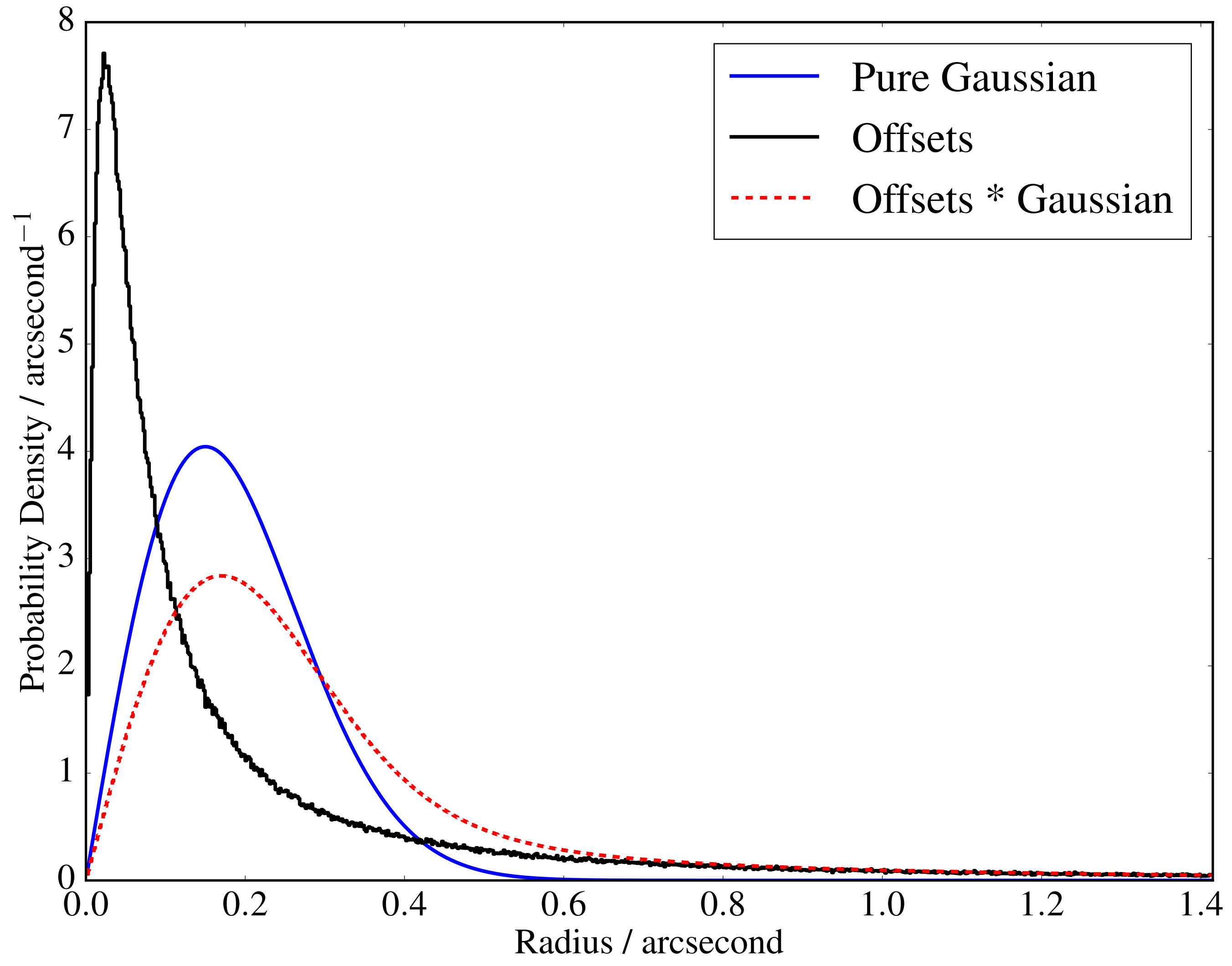


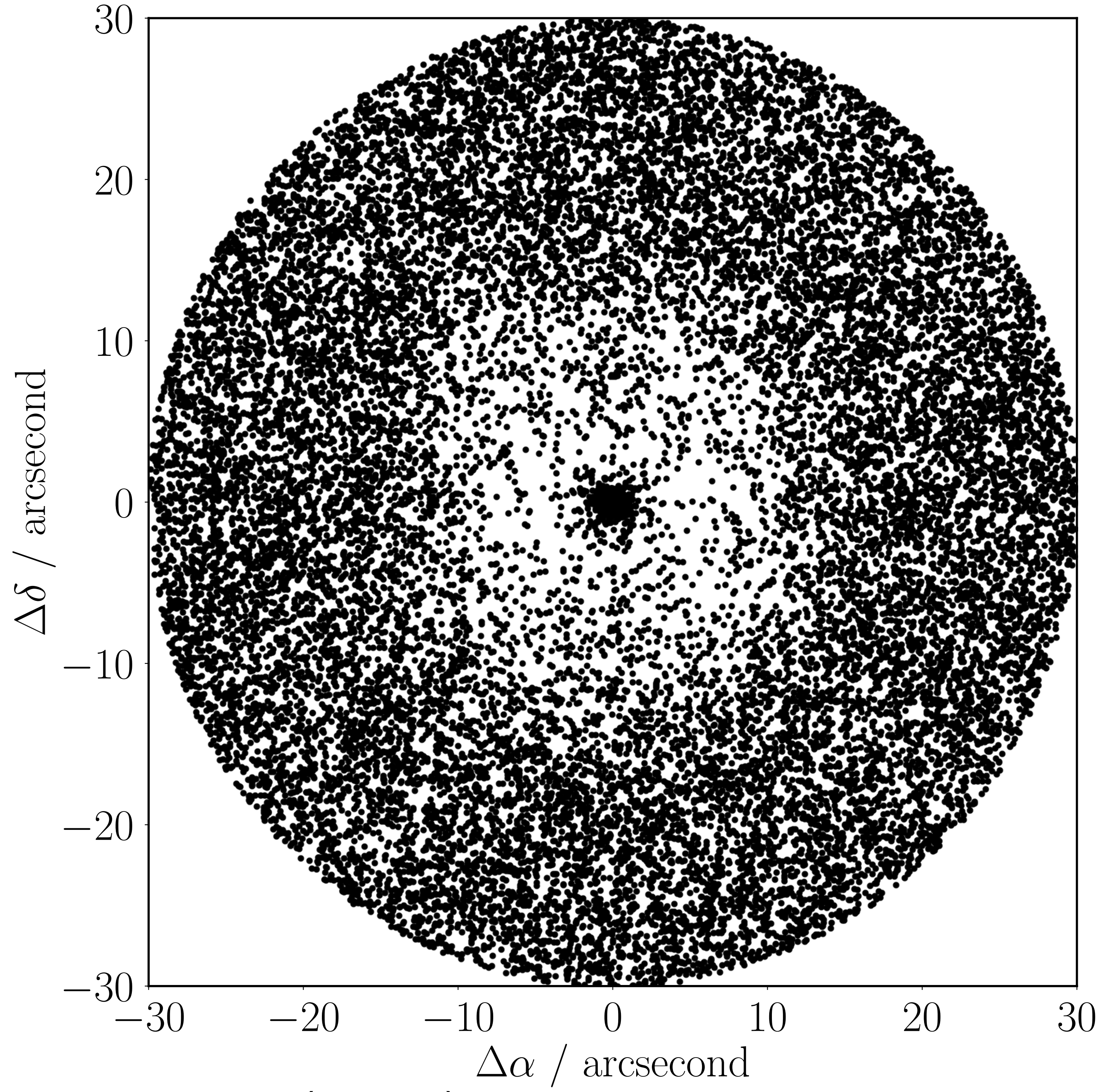

**Without modelling this extra effect, we fail to recover many true pairings, with an artificially high false negative rate!**

*WISE* - Wright et al. (2010)
*Gaia* DR2 - Gaia Collaboration, Brown A. G. A., et al. (2018)






Wilson & Naylor (2018b)




Wilson & Naylor (2017)

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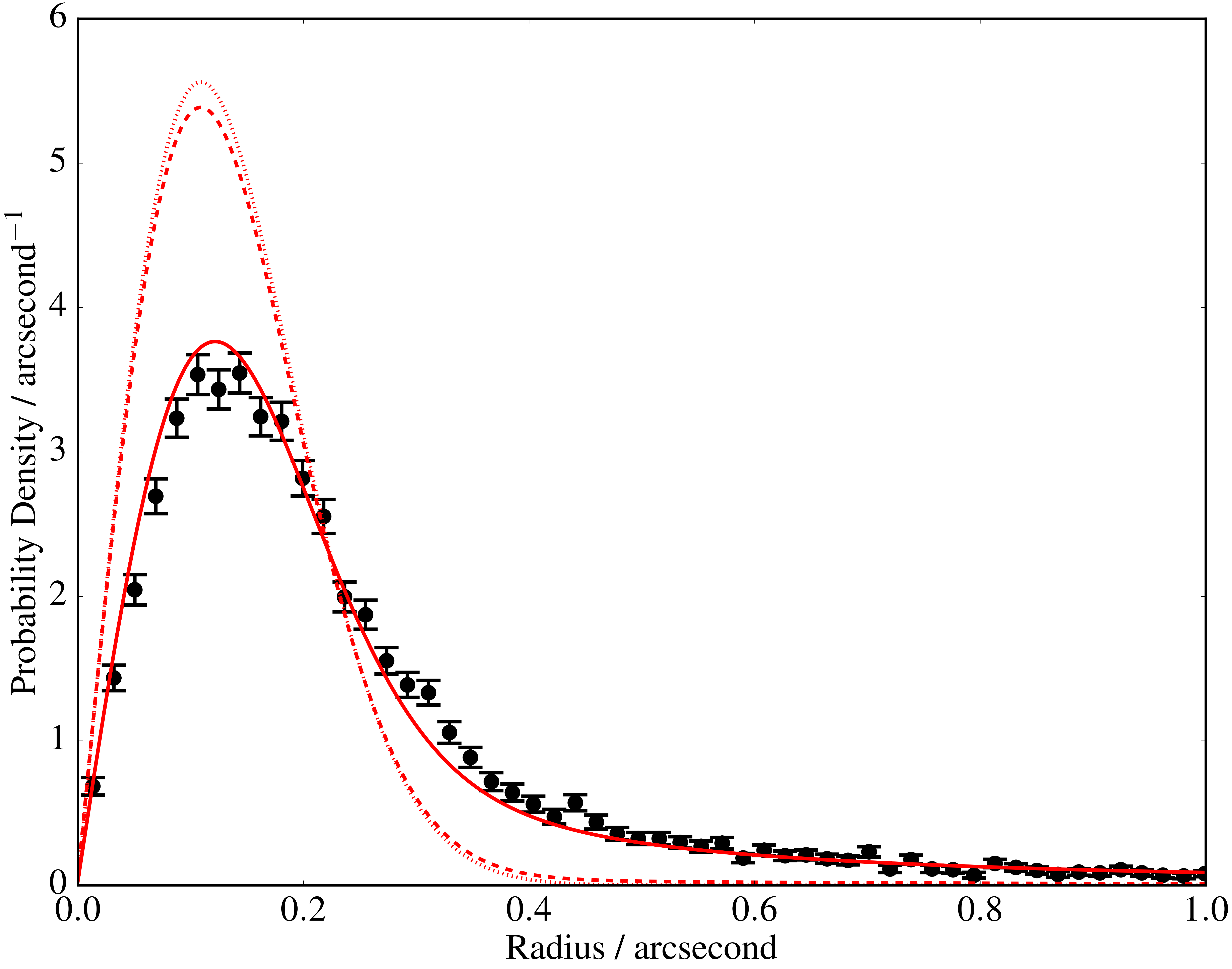
# Extra-galactic Effects of Crowding




Wilson & Naylor (2018b); also see Wilson (2022, RNAAS)

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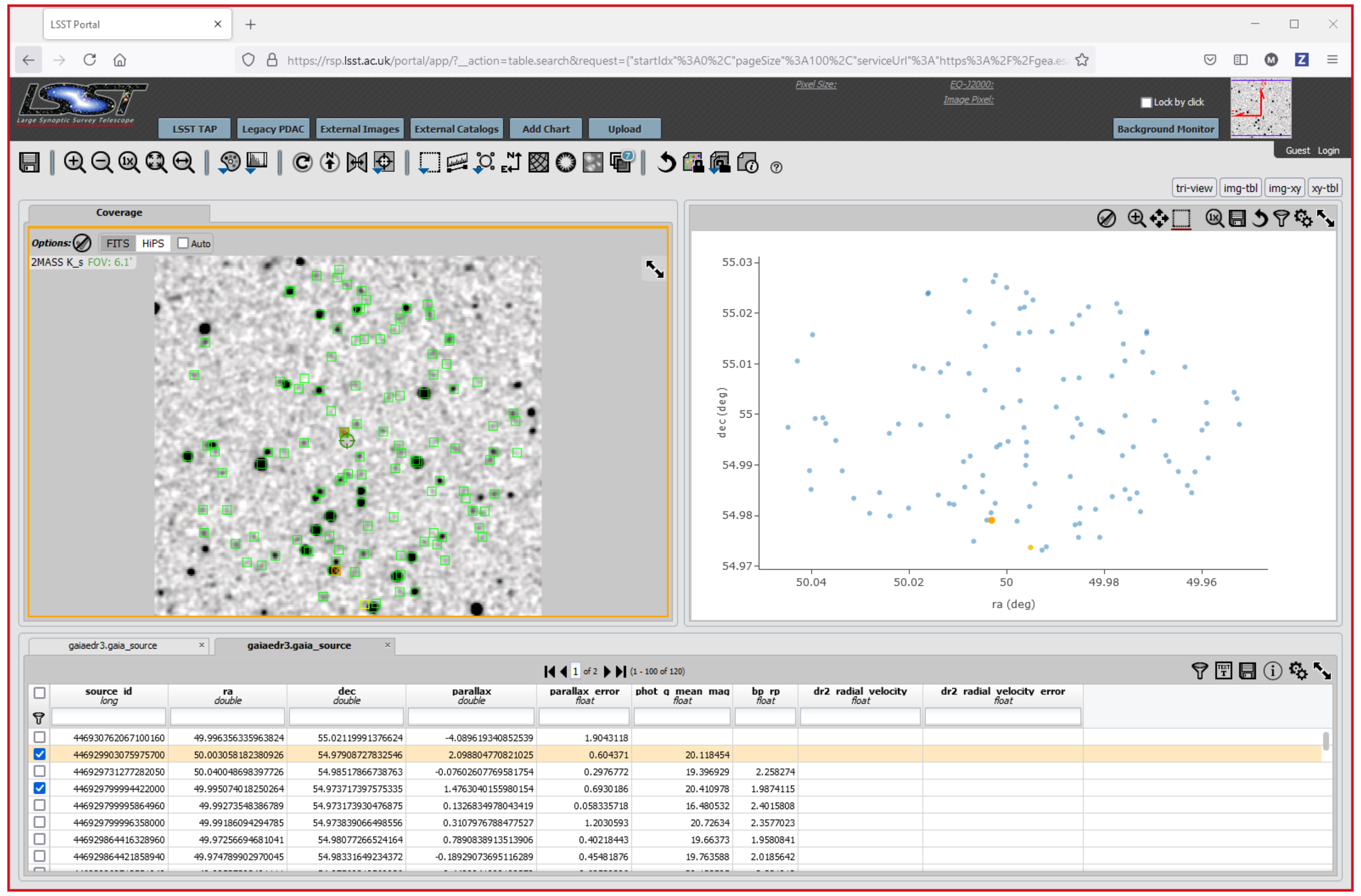
# How To Use Our Cross-Matches

**(Or, how this impacts you on a day-to-day basis)**

**Three tables per cross-match: merged catalogue dataset, and 2x non-match dataset (one per catalogue)**




**Example columns:**

- **Designations of the two sources (e.g., WISE J… and *Gaia* EDR3…)**
- **RA and Dec (or Galactic l/b) of the two sources**
- **Magnitudes (corrected for necessary effects, such as e.g. *Gaia*) in all bandpasses for both objects**
- **Match probability — probability of the most likely permutation (see equation 26 of Wilson & Naylor 2018a)**
- **Eta - Photometric likelihood ratio (counterpart vs non-match probability, just for brightnesses; see eq37 of WN18a)**
- **Xi - Astrometric likelihood ratio (just position match/non-match comparison; see eq38 of WN18a)**
- **Average contamination - simulated mean (percentile) brightening of the two sources, based on number density of catalogue**
- **Probability of sources having blended contaminant above e.g. 1% relative flux**

**We will provide a two match runs per catalogue pair match: one with, and one without, the photometry considered, to allow for the recovery of sources with “weird” colours but otherwise agreeable astrometry**

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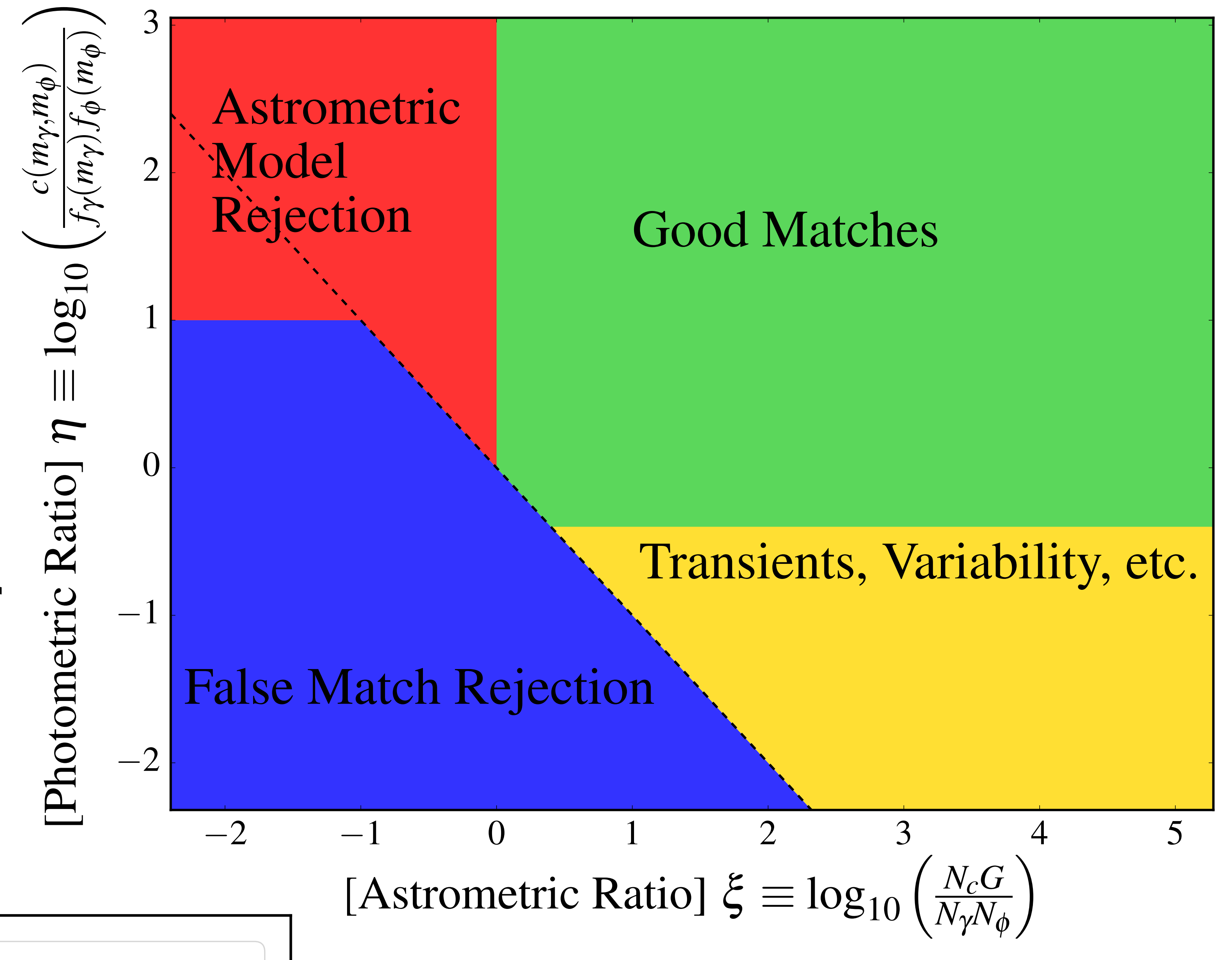
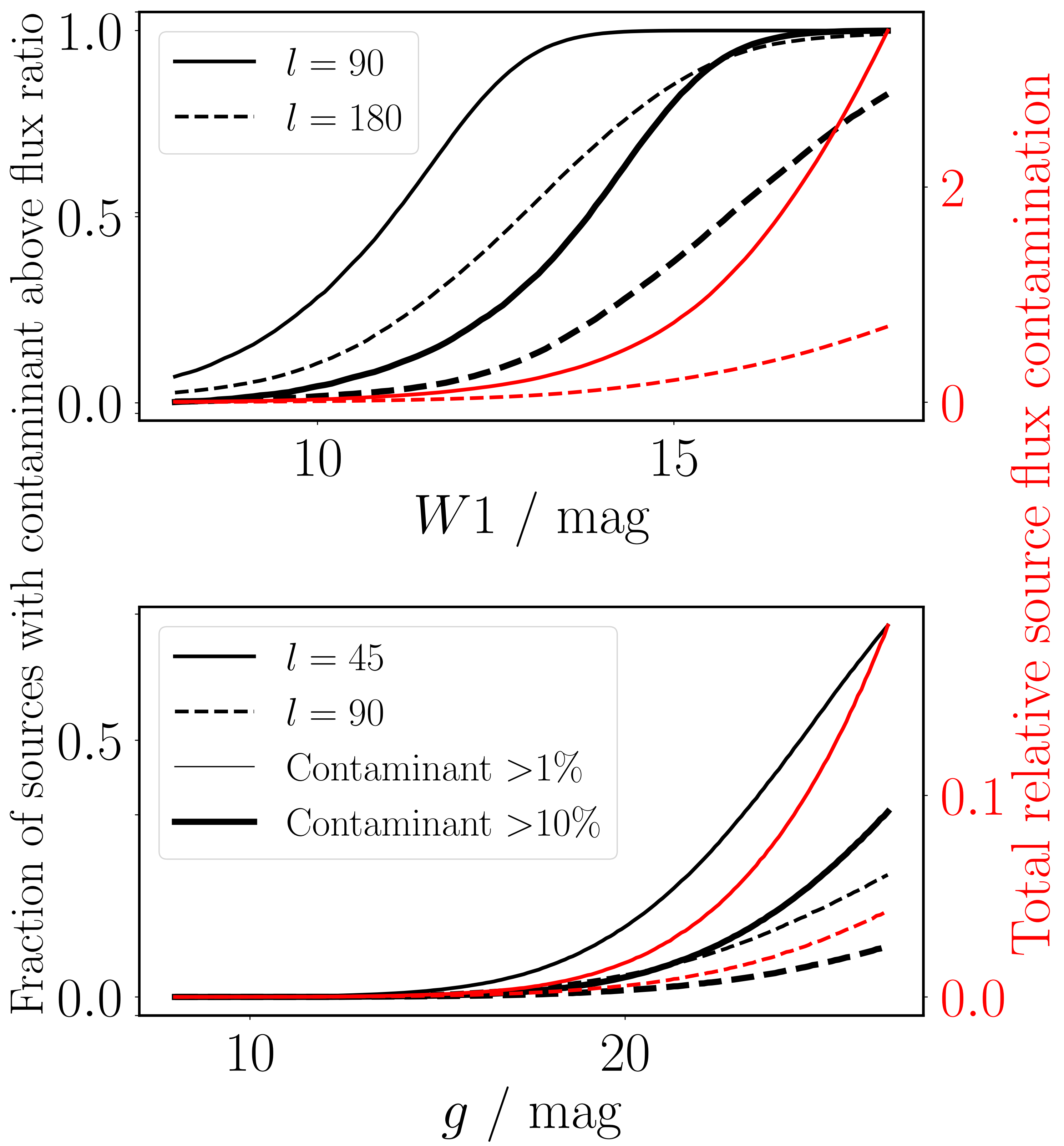
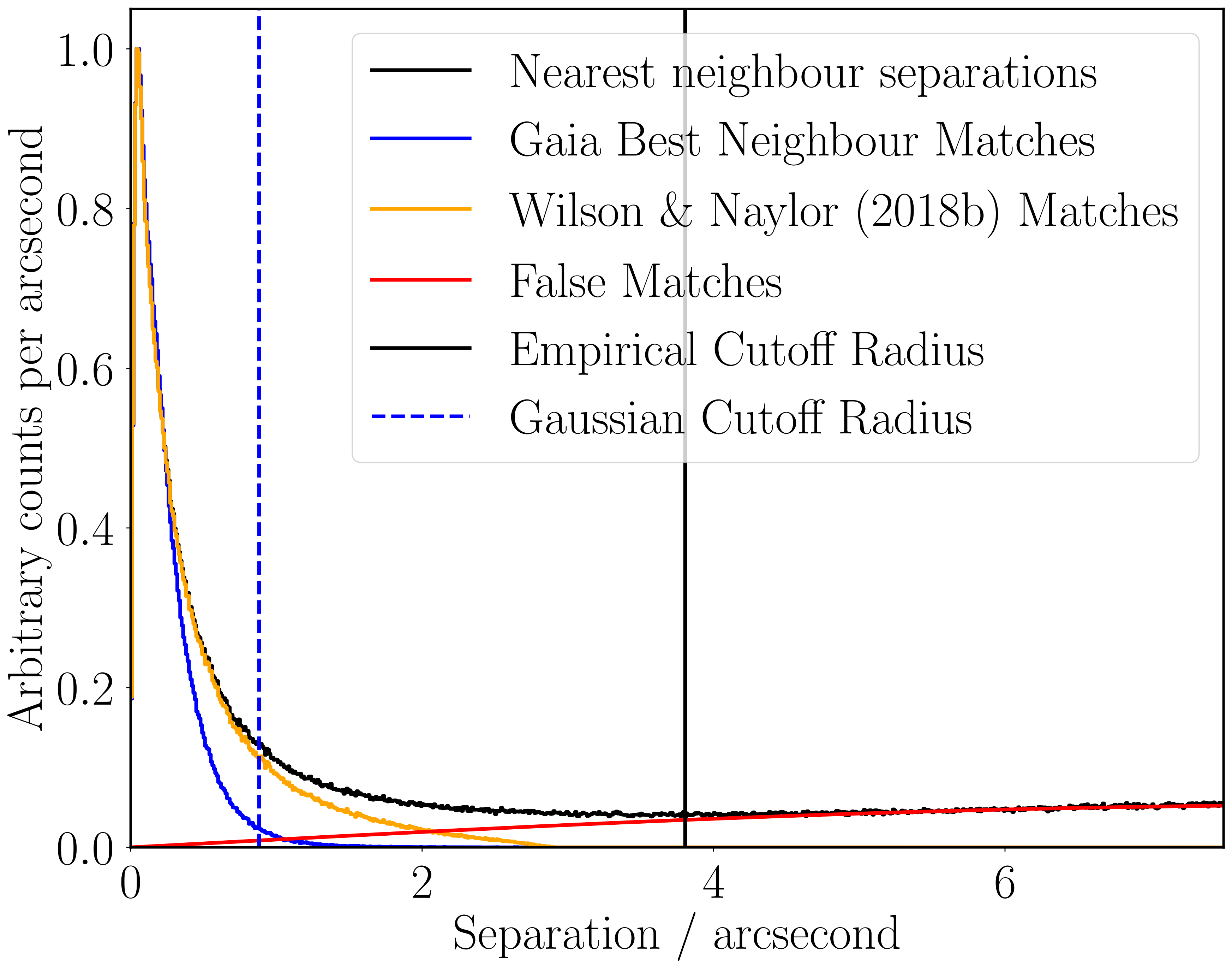
# Why Use Our Cross-Matches?

- **Getting cross-matches, even for "well behaved" fields**
- **Finding "odd" objects, either using the inclusion vs non-inclusion of the photometry in the two match runs, or via the likelihood ratio space — planned "real time" matching service for transient objects**
- **Removing e.g. IR excess or correcting for extinction-like crowding brightening, through Average Contamination; crucial for removing completely unknown crowding of catalogues using aperture photometry**
- **Recovering additional sources missed by other match services — either in crowded fields (we recover up to twice as many *Gaia*-*WISE* matches than the *Gaia* best neighbour matches), or with our in-progress extension to unknown proper motion modelling**

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# Conclusions

- **Upcoming LSST:UK cross-match service macauff — let me know your thoughts/needs/hopes/dreams**
    - **Provide tables of cross-matches between LSST and <your favourite catalogue here!>**

- **Our cross-matches include two key elements for avoiding issues with the crowded LSST sky**
    - **A generalised approach to the Astrometric Uncertainty Function allows for the inclusion of the effects of perturbation due to blended sources and unknown proper motions — reduce false -ves!**
    - **Use of the photometry of sources allows for the rejection of false matches (with >1 “extra” source per 2 arcsecond circle in most of the LSST Galactic plane) — reduce false +ves!**
- **Will include additional information on the crowding of sources, allowing for selection of uncontaminated objects, or modelling of excess flux — crucial for removal of red excess in SEDs**
    - **LSST will suffer of order 10% flux contamination, which could be confused with extinction**

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**Wilson & Naylor, 2017, MNRAS, 468, 2517**
**Wilson & Naylor, 2018a, MNRAS, 473, 5570**
**Wilson & Naylor, 2018b, MNRAS, 481, 2148**
**Wilson (2022, RNAAS)**
**Wilson (2022, RASTI, in review)**
**https://github.com/Onoddil/macauff**

Macauff